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A ficitionalist approach to the ontology of mathematics

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Abstract

Mathematics is defined as the study and development of conceptual interpreted formal systems that are closed by deduction. These systems are not merely syntactic in the manner of logistic systems. They are interpreted, but their reference class is constituted by conceptual artifacts. Mathematical constructs are human-made creations that exist solely within the context of a specific formal system. Consequently, mathematics is devoid of any ontological import. The referents of mathematics cannot exist independently of the human mind. The theory is consistent with any general materialist worldview.

1 Introduction

Mathematics is indispensable for the formulation of scientific theories about the world [1, 2]. However, there is no clear consensus on what kind of objects mathematics refers to. For Platonists, mathematics refers to things like numbers, sets, functions, manifolds, etc. [3]. They are willing to take this reference class at face value and accept the existence of such entities in the world. Nominalists, of course, reject such tolerance of abstract entities [4]. Some claim that mathematics refers to space-time points [5], and others that it does not refer at all [6]. Some materialists claim that mathematical symbols refer to themselves [7], i.e. to physical inscriptions. The variety of positions is wide, diverse, and somewhat confusing.

I declare myself to be a materialist. I do not accept the existence of ghosts, God, numbers, sets, functions, manifolds, etc. in the world. Nevertheless, I think that mathematics refers to some of these objects. To reconcile these claims in a coherent theory of mathematical ontology is the manifest purpose of this paper. The less obvious goal is to outline a philosophy of mathematics and to answer some criticisms that might be made of these views. I make no claim to originality. Similar ideas have been presented by several authors, most notably Vaihinger [8], Bunge [9, 10], Curry [11], Bueno [12], Woods [13] and myself [2]. However, the approach I will present and some details can claim novelty. Of the cited authors, only Bunge considered himself a materialist. My ultimate goal is to offer the scientist a theory of mathematics that is fully compatible with a materialist worldview [14].

I will begin with some preliminary remarks on formal languages, since I think of mathematics as a family of formal conceptual systems in the tradition of David Hilbert, Haskell Curry [11], and Alan Weir [15].

2 Preliminaries: formal languages and mathematics

Languages are conceptual and symbolic systems used for communication and representation. It is common to divide languages into *natural* and *formal* languages. Natural languages are the result of biological and cultural evolution. They play the role of a tool for communication. Because of how they evolved, they can be vague and imprecise. Translation between natural languages developed

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in different socio-cultural contexts is difficult, if not impossible, because of semantic indetermination [16, 17]. Formal languages, on the other hand, are designed to be free of these problems. They have explicit rules for the formation of valid terms based on a primitive vocabulary that is also explicit.

2.1 Formal languages

A formal language is a conceptual system equipped with a set of rules for generating valid combinations of symbols. A formal language L_1 is expressed in a metalanguage L_2, which can be formed by elements of L_1 and other languages (including natural languages), to avoid the formation of paradoxes. We can represent a formal language L_1 as a triplet:

$$L_1 = \langle \Sigma_{L_1}, R, \Omega \rangle . \tag{1}$$

Here Σ_{L_1} is a set of language primitives, R is the set of rules that provide explicit instructions on how to form valid combinations of elements of Σ_{L_1}, and Ω is the set of objects denoted or designated by the elements of L_1. The set R is made up of three disjoint subsets:

$$R = S_{\text{sy}} \cup S_{\text{se}} \cup Pr, \tag{2}$$

where S_{sy} is a set of syntactic rules, S_{se} is a set of semantic rules, and Pr is a set of pragmatic rules. The first set consists of rules for forming terms that are allowed in the language, the second set consists of rules that relate terms of L_1 to objects of the discourse domain Ω, and Pr are pragmatic rules that should be adopted for the correct use of the language in a given context.

In addition to these rules, inference rules are usually added. The most common ones are:

- *Modus ponens* (MP): If $A \wedge (A \rightarrow B)$, then B.
- *Generalization rule* (Gen): If A, then $(\forall x)A(x)$, where x is a variable. In these inference rules, all the symbols are those usual in first order logic.

Definitions can also be built into the language to simplify notation and reduce the complexity of the various terms that will appear in the language. A definition is the elucidation of a new symbol in terms of other primitive symbols.

The operation of deduction makes it possible to obtain valid formulas from other valid forms. Deduction is the successive application of syntactic rules; the formulas obtained by deduction are called theorems. We use the symbol $\vdash$ to mean "this is a theorem". In particular, we say that a system S of formulas is *consistent* if and only if $\neg(S \vdash \phi \wedge \neg\phi)$ for all $\phi \in S$. The formula $\phi \wedge \neg\phi$ is called a *contradiction* and is usually excluded from formal languages because $\phi \wedge \neg\phi \vdash B$, where B is anything. In most cases we are interested in languages that are consistent, especially when they are used to represent extralinguistic objects. This is because things and events are not contradictory. They just are what they are. Contradiction can only occur in our languages.

If a formal language is such that $S_{\text{se}} = Pr = \emptyset$, then the language is a *logistic system*. Logistic systems, or abstract languages, are purely syntactic. They do not refer to anything, because referring is a semantic relation.

2.2 Mathematics as a family of formal languages

Mathematics, instead, are a family of formal languages that clearly refer to entities such as numbers, sets, functions, manifolds, equations, etc. These objects, however, are carefully constructed in the framework of the corresponding mathematical theories. We shall call them, therefore, *formal constructs* or, more generally, *conceptual artifacts*. An artifact is something constructed by an intentional agent with a given purpose. If the artifact is conceptual, then its components and links are not material but created by a set of stipulations within a formal system. More specifically, we define:

Definition: An object x of a consistent formal system S is a *conceptual artifact* if, and only if, there is a well-formed set C such that $x \in C$ and C is specified in S.

C includes a subset of formation rules in S. Torretti [18] argues that this definition is circular because it invokes a conceptual artifact: the set C. But for every set C there will be another formal system S' such that $C \in C'$. What, then, about the set of all sets of conceptual artifacts? Since such a set cannot be formulated consistently [19] (i.e. it is ill-formed), it is not a conceptual artifact. Then, there is no circularity in our definition. Conceptual artifacts can only be introduced by their characterization in *consistent formal theories*.

We can now define mathematics as follows [2]:

Definition M1: *Mathematics* is the set of all *mathematical systems*,

where,

Definition M2: A *mathematical system* is a consistent formal system such that the semantic rules relate symbols in the system with conceptual artifacts, which are formed in accordance to constitutive rules that are exempt from any vagueness or ambiguity.

A statement in a mathematical system will be true if, and only if, the statement can be proved within the system, i.e. P is true in S iff $S \vdash P$. Then, truth, in mathematical theories, is equivalent to theoremhood.

Formal systems, and hence mathematics, are human creations. They are exact because the precise formation rules of their formulas are made explicit. They are conceptual because they are independent of any material object (with the exception of the human beings that create them). Formal systems exist only to the extent that there are people capable of thinking them. This does not mean they are subjective. They are perfectly impersonal and intersubjective. Any person can use the formation rules of the system to check the validity of all statements. Mathematics, therefore, is exempt from the vagueness that plagues natural languages. This is why mathematics is so useful for factual sciences: it provides an exact and unambiguous framework to express our ideas about the world.

2.3 Reference and meaning in mathematics

The meaning of mathematical terms is a two dimensional concept that we attribute to the term after the analysis of the particular system where the term occurs. Let us call T to a specific mathematical theory. A theory is a set of statements such that the set is closed under the operation of deduction, i.e. $T = \langle A, \vdash \rangle$, where A is a set of independent statements called axioms. Given a term x in a theory T, we define the meaning of x (denoted by $M_T(x)$) as

$$M_T(x) = \langle \mathcal{R}_T(x), \, \mathcal{S}_T(x) \rangle. \tag{3}$$

Here, $\mathcal{R}_T(x)$ is the *reference* of x in T and $\mathcal{S}_T(x)$ is the *sense* of x in T [20, 21, 2]. The reference is a relation between the term (symbol or chain of symbols) and a conceptual artifact. In the case of a predicate, it can be defined as the set of all its arguments. To illustrate, the assertion 'real numbers are commutative with respect to multiplication' refers to the domain of real numbers. It should be noted that the term "reference" is distinct from "extension". The statement '$\neg\exists x(x \,\text{is prime} \wedge 8 < x < 10)$' has an empty extension but refers to prime numbers.

In regard to the sense of a mathematical construct, it is defined as the union of all the items in a theory that entail or are entailed by it. In symbols, we have that $\mathcal{S}_T(c) = \{x : x \vdash c\} \cup \{y : c \vdash y\} = A_T(c) \cup J_T(c)$, where $A_T(c)$ is the purport or logical ancestry of c and $J_T(c)$ is the import or logical progeny of c in T.

It is my contention that mathematics is not, as some have suggested, a meaningless collection of symbols or inscriptions. Rather, it is a system of formal theories whose terms have a well-defined meaning. This is why we can comprehend them, grasp their referents, and discern their implications. Does this entail that I accept the existence of conceptual artifacts such as Hilbert spaces, numbers, or sets? No, it does not.

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3 What is 'to exist'?

In any discourse, there are two types of commitments: quantifying commitment and ontological commitment. Quantifying commitment is made to objects that are within the range of the variables of our quantifiers. Ontological commitment is incurred when we commit to the factual existence of certain objects. However, despite Quine's view [22], quantifier commitment does not involve ontological commitment. This can be shown by distinguishing between partial quantifiers and existence predicates. The objective is to refrain from interpreting the existential quantifier as carrying any ontological commitment. In contrast, the existential quantifier merely indicates which objects fall under a given concept (or possess certain properties). The objects in question constitute a subset of the entire domain of discourse. To indicate that the entire domain is invoked (i.e., that every object in the domain has a certain property), a universal quantifier is employed.

In the traditional "Quinean" reading of the existential quantifier, two distinct functions are grouped together: (i) to affirm the existence of something, on the one hand, and (ii) to indicate that the entire domain of quantification is not considered, on the other hand. It is preferable to maintain these functions separated. I recommend that a partial quantifier (i.e., an existential quantifier devoid of ontological commitment) be employed to indicate that only a subset of the objects within the domain are being referred to. Then, an existence predicate can be introduced to express existence assertions. By distinguishing these two roles of the quantifier, it is possible to gain expressive resources.

Let us suppose that "$\exists$" is used to denote the partial quantifier and that "E" is used to denote the existential predicate. In this case, we can express the following: $\exists x\,(Fx \wedge \neg \mathrm{E}x)$. This can be interpreted as "some objects have property F and those objects are not real" (i.e. do not exist).

Logical quantification merely establishes a correspondence between individuation and formal coherence,

$$\exists x\, f(x) \leftrightarrow \{x : f(x)\} \neq \emptyset, \tag{4}$$

where $\emptyset = \{x : x \neq x\}$ is the empty set. This means that formal existence implies nothing more than being free of contradictions. If we abstain in mathematics of using the existential predicate "E", we shall remain free from ontological commitments. When we refer to something in mathematics, we refer to a consistently defined conceptual artifacts. But we are not saying that such artifact exists in the world, independently of the formal system where it is introduced.

4 Formal existence is contextual

Since truths about mathematical artifacts always depend on the corresponding formal context in which they are defined, mathematical truths are analytic and independent of the world (although mathematical statements are not tautologies, as Wittgenstein and his followers once thought). For example, if we introduce an operator $\mathcal{T}^M$ that expresses "the proposition (...x...) is true in the system M", we can write that

$$(\exists x)\, \mathcal{T}^M(...x...). \tag{5}$$

If M is the theory of integers, an instance of this could be "it is true that there exists an integer less than 4 and greater than 2". But this does not imply that there is a material object called "3". This proposition is true in M, not in the world. Existence in a formal systems is contextual, i.e. it is valid only within the system where the formation rules were introduced.

5 Conceptual references are fictional

Fictionalism is a view of the nature of mathematical objects. The central point of the fictionalist approach is to emphasize that mathematical entities are fictional entities. They have attributes similar to those of fictional characters such as Sherlock Holmes or Hamlet. The fictionalist's proposal is to view mathematical objects as conceptual artifacts. Artifacts of any kind, material or conceptual, are human creations. In the case of conceptual artifacts, they do not actually exist, they do not have causal power, they do not interact with material objects, they are not located in spacetime, and so on.

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In other words, they are not real in the sense that they are fictional objects created by the intentional acts of their authors. Thus, they are introduced in a particular context, at a particular time. However, unlike fictional characters in stories, mathematical artifacts are not free creations, but are constrained by strict rules established in the context of a mathematical theory. They are fictions, yes, but of a very special kind: mathematical entities are created when constitutional principles are proposed to describe their place and function in a formal system, and when consequences are drawn from those principles. What they share with fictional entities is that we refer to them as if they were real, assuming that it is understood that we are referring to a formal context in which they are defined. This context is created by mathematicians through the activity of their brains and presented to other people in publications, conferences, and electronic devices.

The mathematical entities thus introduced also depend on (i) the existence of particular copies of the works in which the constituent principles were presented (or memories in particular individuals of those works), and (ii) the existence of a community capable of understanding them. It is correct to say that the mathematics of a particular community has been lost if all copies of its mathematical works have been lost and there is no memory of them. Of course, if other mathematicians reintroduce the same formation rules and adopt the same inference rules, the context and the corresponding artifacts will be recreated so that we can refer to them again. This has happened many times in the history of mathematics. I think, for example, of Pascal's rediscovery of Euclidean geometry, or Heisenberg's rediscovery of matrix algebra.

Thus the mathematical entities introduced by the relevant constitutive principles turn out to be contingent, at least in the sense that they depend on the existence of certain concrete objects in the world, such as human beings and their mathematical works. They do not exist independently of the people who invent them. Fictionalism is a materialist theory of mathematics because it does not postulate abstract entities independent of the human mind.

The fictionalist insists that there is nothing mysterious about how we can refer to mathematical objects and have knowledge about them. Reference to mathematical objects is made possible by works in which the relevant constitutive principles are formulated. In these works the corresponding mathematical objects are introduced. The principles specify the meaning of the mathematical terms as well as the properties of the mathematical objects. In this sense, the constitutive principles provide the context in which we can refer to and describe the mathematical objects in question as if they were real.

Our knowledge of mathematical objects is obtained by examining the attributes of these objects in the context in which they are incorporated, and by drawing consequences from the principles according to which we introduce them into formal systems. It does not require any special intuition about an abstract world.

6 Discussion on objections

The main criticism of the philosophical approach I have presented here has been made by J.-P. Marquis, in his reply to Bunge's version of fictionalism [23, 24]. One of the points raised by Marquis is "What distinguishes mathematical conceptual systems?". He claims that Bunge's characterization of mathematics based on three main requirements [1] can also be applied to, for instance, metaphysics:

> One could argue that there are large parts of philosophy, at least as it is traditionally conceived, that satisfy these three properties. For instance, metaphysics is purely conceptual; philosophers posit and conjecture general patterns and some philosophers try to prove or disprove some of their conjectures. In fact, Bunge's own work in ontology seems to fall under this characterization. I, for one, am far from being satisfied by this enumeration. [23].

[1] "What makes the mathematical study of conceptual systems unique is that (a) it is purely conceptual (i.e. does not make essential use of any empirical data or procedures) and it involves, at some point or other, (b) positing or conjecturing the laws (general patterns) satisfied by the members of those conceptual systems, as well as (c) proving or disproving conclusively some such conjectures." [9]

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I do not think that metaphysics is purely formal, as Marquis claims (see [2]), but is informed by the special sciences, such as physics or biology. However, the criticism does not apply to my approach, since I do not claim to characterize mathematics as a family of formal systems from Bunge's desiderata. Rather, I think that what distinguishes mathematics as a formal system from other conceptual constructions or systems is that mathematics is *exact* (in the sense that it is free of vagueness[2]), *consistent* (in the sense that it does not contain contradictions), has both *syntactic* and *semantic rules* for the formation of valid expressions along with inference rules, and never refers to entities supposed to exist independently of the formal system.

Marquis raises a second objection in ref. [23] to the claim that conceptual artifacts are fictions. He states:

> Why is it that mathematical ideas, creations of the human brain, do not have properties linked to this creation? Why don't they have historical properties, reflecting some peculiar socio-historical aspects of the society in which they were created? Or properties of the mathematicians, of their personality? Why don't they have neurophysiological properties? [23]

The answer is that mathematicians construct mathematical systems and theories in this way, from constitutive principles devoid of any reference to historical, personal, cultural, or psychological features. Once the constitutive rules are fixed, the mathematician's work is to determine what follows from them. Since reference is invariant under the operation of deduction, it is irrational to expect historical or psychological aspects to appear in the theorems or definitions of subsequent derivations. Bunge himself is very clear on this point.

> Far from being totally free inventions, mathematical objects are constrained by laws (axioms, definitions, theorems); consequently they cannot behave "out of character" – e.g., there can be no such thing as a triangular circle, whereas even mad Don Quixote is occasionally lucid. [10]

Marquis goes even further, asserting that mathematical constructs are not free creations but are, in fact, conditioned by our cognitive biases, which were formed in our natural and social evolution.

> Indeed, if our numerical and geometrical abilities lie in neurological systems that are independent of language, we can expect that our experience of this knowledge will have a quality that goes beyond language and, in some sense, explicit consciousness. This would account in part for our feeling that mathematics is something that lies beyond and behind our conscious experience, that it is something that we discover. And, indeed, in a very specific sense, we do. [23]

Our cognitive abilities developed by evolution undoubtedly favor brain operations such as counting and grouping. This in turn inspires mathematicians to formulate certain constitutive rules or axioms. Once they are imposed and a system is formed, all discoveries are related to the implications of the rules in the system. It is a simple fact that no mathematical artifact has ever existed independently of a human brain that thought about it. Furthermore, no mathematical artifact has been autonomous or changed of properties (attributes) within that system. Of course, other human beings could explore other systems, obtained from the first one through modifications of the rules. This is evidenced by the development of non-Euclidean geometries from the Euclidean one, and other similar examples.

In a more recent work, Marquis [24] directly challenges Bunge's assertion that mathematical artifacts are mere fictions, existing only in a given formal context but not in reality. If a conceptual object, such as a mathematical artifact, is to be defined within the context of a mathematical system, then the system itself must be considered. Is it real or a mere fiction? We have argued that mathematical systems are formal systems where semantic rules connect conceptual artifacts with symbols of the system. Marquis would argue, perhaps, that some forms of art also do the same. That is true, but

[2] For a formal definition of vagueness, see my book *Scientific Philosophy*, chapter 2 [2].

mathematical systems, contrary to artistic systems, are consistent and exact. In art, interpretation is often ambiguous and consistency is not necessarily a requirement. In fact, much of the expressive power of art is based on such an ambiguity.

Marquis then advocates for a structuralist approach. He boldly asserts that "mathematics is about structures". In his own words:

> For a mathematical theory to be a structuralist theory, it should be possible to prove that the following claim is a metatheorem: given any property P in the given language $\mathcal{L}$ of the theory T, for all objects, of the theory, if $P(X)$ and $X \cong Y$, then $P(Y)$. In words, a theory is a structuralist theory if the provable properties of the theory are only those that are invariant under the proper notion of isomorphism. This says precisely that mathematics is about the properties and relations expressed in the proper language and that the underlying objects merely fill in the places to be filled in the relations of the theory. The specific nature of the objects is totally irrelevant. It is in this sense that mathematical objects are not the central concern and that they are always part of a system [24].

While Marquis does not explicitly state this, it is likely that the symbol $\cong$ represents the concept of "isomorphic to". This structuralism is perhaps a viable approach for abstract algebras, but not for any interpreted mathematical system. Mathematics is concerned with the study of fundamental concepts such as the number 4, the ratio of the circumference of a circle to its diameter (π), the exponential function, and geometrical objects as triangles or spheres. It is not in accordance with the tenets of mathematical practice to downplay the significance of such entities. Conversely, in physics, invariant properties and covariant law statements are the norm. This suggests that physics should be regarded, in Marquis view, as a subfield of mathematics. Nevertheless, the majority of physicists would be reluctant to espouse such a perspective.

7 Conclusions

I have presented a materialist theory of mathematics based on an understanding of mathematics as a family of formal systems that are interpreted only conceptually, in the sense that they refer excessively to conceptual artifacts. These artifacts are human creations, but not completely free creations, since they are subject to strict constitutive rules introduced into the system that make them exact, i.e. free from vagueness. Mathematical objects share with fictional creations the fact that they are creations of the human brain with no independent existence, although, unlike literary fictions, they are necessary within a closed system once the constitutive principles have been made explicit. The main points of this view can be summarized as follows:

(1) Mathematical knowledge: Understanding and thus knowledge of mathematical entities, as well as knowledge of fictional entities in general, is the result of producing adequate descriptions of the objects in question and drawing consequences from the assumptions made to define them.

(2) Reference to mathematical objects: How is reference to mathematical objects accommodated in the fictionalist's approach? The formative principles adopted specify the attributes of the mathematical objects to be introduced. It is possible to refer to the objects in question as those objects that have the corresponding attributes. Mathematical reference is always contextual: it is made in the context of the constitutive principles that give meaning to the relevant mathematical terms.

(3) Application of mathematics: For the fictionalist, the application of mathematics to reality is a matter of using the expressive resources of mathematical theories to accommodate various aspects of scientific discourse. The only requirement is that the mathematical theory be coherent, that is, free of contradictions. Thus, the criterion of truth in mathematics is internal coherence.

To conclude: Mathematics can be understood as the study and development of fictitiously interpreted formal systems that are closed by deduction. These systems are not purely syntactic like logistic systems. They are interpreted, but their reference class consists of exact conceptual artifacts. They are mental constructs produced by human beings that exist only in the context of a given formalism into which they are introduced. Therefore, mathematics has no ontological import. The referents of mathematics cannot exist independently of the human brain.

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