Journal homepage http://revistas.unitru.edu.pe/index.php/SSMM

SELECCIONES MATEMÁTICAS
Universidad Nacional de Trujillo
ISSN: 2411-1783 (Online)
2023; Vol. 10(2): 249-256.

A mathematical model for the formation of oscillation marks in steelmaking industry

Marcos Zambrano Fernández

Received, Set. 31, 2023; *Accepted, Oct. 20, 2023;* *Published, Dec. 27, 2023*

How to cite this article:
Zambrano M. *A mathematical model for the formation of oscillation marks in steelmaking industry*. Selecciones Matemáticas. 2023;10(2):249–256. http://dx.doi.org/10.17268/sel.mat.2023.02.02

Abstract
Oscillation marks are defects found on the surface of the final product in the steelmaking industry. In this research, the temperature of a lubricant known as flux that lies inside of the mould is investigated due to its importance in the formation of the oscillation marks. This temperature that decreases in time is obtained numerically via the finite-volume method by using the Openfoam software.
Keywords . Oscillation mark, steelmaking industry, flux, finite volume method.

1. Introduction. In steelmaking industry, there are two process usually employed to produce solid steel: The ingot and the continuous casting. The ingot casting is a traditional technique that comprises the solidification of the liquid steel confined in a static mould and subsequently is extracted to obtain the final product as solid steel.

Some decades ago, a new technique called continuous casting has emerged aiming to produce more amount of solid steel by passing continuously the liquid steel through a mould. In figure 1.1 is shown the continuous casting process wherein blocks of solid steel are obtained by exiting from the mould.

Figure 1.1: Steelmaking industry.

*Departamento de Ciencias Básicas - Universidad Nacional de Barranca - Perú. **Author correspondence:** (mzambrano@unab.edu.pe).

A defect found in the final product in the continuous casting process is given by the existence of small indentations on the surface of steel [1]; this indentation is commonly referred to as **oscillation mark** (OM). In figure 1.2 can be observed the formation of OM's on the surface of the cast steel.

As pointed out by Vynnycky *et al.* in [2, 3], the formation of OM's is a consequence of the oscillation of the mould whereby the liquid steel passes continuously. This oscillation is mathematically expressed as

$$v_m(t) = v_0 \cos(\omega t), \tag{1.1}$$

where v_0 is the magnitude of the oscillation and ω its frequency.

Likewise, to improve the process of continuous casting a lubricant powder is added at the top of the mould which becomes liquid inside of the mould and lubricates the mould wall adjacent to the steel. This lubricant is usually referred to as **flux** and the formation of an OM is related to the temperature of the flux during the casting operation.

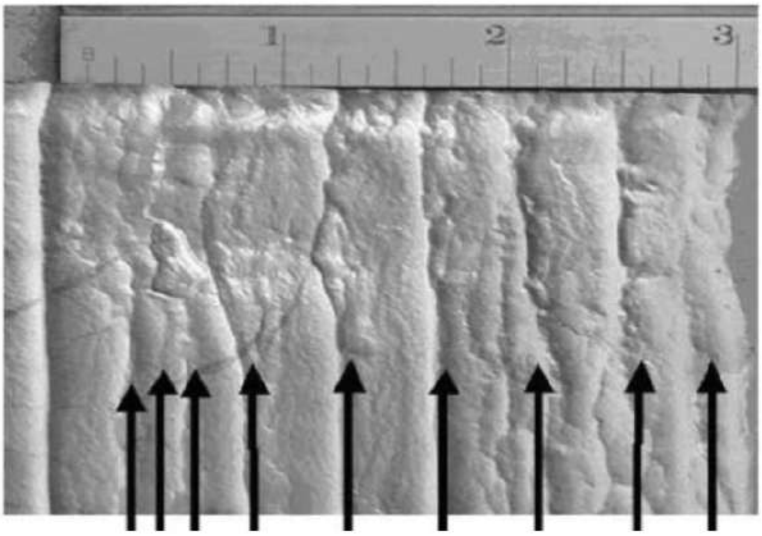

Figure 1.2: Oscillation marks on the surface of the solid steel.

In figure 1.2 are shown OM's which are filled up with the flux initially in liquid state and subsequently by the cooling effect at the mould wall becomes in solid state.

2. Mathematical model. Mathematical modelling of a continuous casting process includes the formulation of the fluid flow and heat transfer equations [4] and hence it is worthwhile to consider them for the formation of an OM that is formed between the mould wall and the solid steel as depicted in figure 2.1 and its profile is given by

$$x = h(z, t). \tag{2.1}$$

Likewise, for the fluid flow of the flux, u, v denote the components of the velocity in the horizontal and vertical coordinates, x, z, respectively and this fluid flow is ruled by the equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0, \tag{2.2}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \rho_f\, g, \tag{2.3}$$

subject to the boundary conditions,

$$u = 0,\ v = v_m(t), \tag{2.4}$$

at $x = 0$ and at $x = h(z, t)$,

$$u = 0,\, v = v_c, \tag{2.5}$$

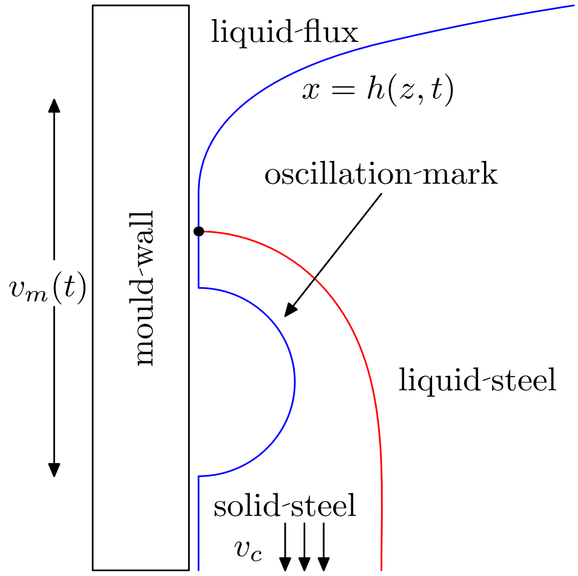

Figure 2.1: Schematic of the process of formation of an OM, which lies between the mould wall and the solid steel.

where v_c is the velocity at which the solid steel is being extracted from the mould usually known as **casting velocity** and μ is the viscosity of the liquid flux.

Also, the heat transfer equation for the liquid flux in the mould is ruled by

$$\rho_f\, c_p \left(\frac{\partial T}{\partial t} + u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{2.6}$$

where ρ, c_p and k are the density, heat capacity and thermal conductivity respectively; and the boundary conditions are given by

$$k\,\frac{\partial T}{\partial x} = m\,(T - T_w), \tag{2.7}$$

at $x = 0$, where T_w and m are the water temperature and the convective heat transfer coefficient adyacent to the mould wall; also, at $x = h(z,t)$,

$$T = T_s, \tag{2.8}$$

where T_s is the temperature at which the steel solidifies.

3. Analysis and numerics.

Using the nondimensionalization

$$X = \frac{x}{\alpha^{1/2}[t]^{1/2}},\; Z = \frac{z}{v_c\,[t]},\; \tau = \frac{t}{[t]}, \tag{3.1}$$

$$U = \frac{u}{\alpha^{1/2}/[t]^{1/2}},\; V = \frac{v}{v_c},\; \theta = \frac{T - T_w}{T_s - T_w},\; H = \frac{h}{\alpha^{1/2}[t]^{1/2}}, \tag{3.2}$$

where $\alpha = \rho/k\,c_p$ is the thermal diffusivity and

$$[t] = \frac{2\pi}{\omega}, \tag{3.3}$$

is a length scale for time. Hence, equation (2.6) becomes

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial X^2} + \varepsilon\,\frac{\partial^2 \theta}{\partial Z^2}, \tag{3.4}$$

where

$$\varepsilon = \frac{\alpha}{v_c\,[t]}. \tag{3.5}$$

and the boundary conditions for equation (2.6) are given by

$$\frac{\partial \theta}{\partial X} = \beta\, \theta, \tag{3.6}$$

at $X = 0$ from (2.7), where β is defined by

$$\beta := \frac{m\, \alpha^{1/2} [t]^{1/2}}{k}, \tag{3.7}$$

also at $X = H(Z, \tau)$

$$\theta = 1. \tag{3.8}$$

from (2.8). Likewise, Vynnycky *et al.* in [3] found that the profile of an OM in nondimensionalized coordinates is given by

$$H(Z, \tau) = \left\{ 3\, \frac{\mathcal{V}_0 \cos(\tau - Z) - 1}{\Gamma - \Lambda} \right\}^{1/2}, \tag{3.9}$$

where Γ and Λ are defined respectively by

$$\Gamma = \frac{\rho_s\, g\, \alpha\, [t]}{v_c\, \mu}, \quad \Lambda = \frac{\rho_f\, g\, \alpha\, [t]}{v_c\, \mu} \tag{3.10}$$

and

$$\mathcal{V}_0 = \frac{v_0}{v_c}, \tag{3.11}$$

is the magnitude of the mould velocity in nondimensionalized coordinates. This mould velocity now is given by

$$\mathcal{V} = \mathcal{V}_0 \cos(2\pi\tau). \tag{3.12}$$

Symbol	**Meaning**	**Value**	**Units**
ρ_f	Density of the flux.	2930	kg m^{-3}
ρ_s	Density of the liquid steel.	7800	kg m^{-3}
k	Thermal conductivity.	1.5	W m^{-1}k^{-1}
c_p	Heat capacity.	1260	J kg^{-1}K^{-1}
m	Convective heat transfer coefficient.	10000	W m^{-1}k^{-1}
μ	Viscosity of the flux.	0.5	Pa s
v_0	Amplitude of the mould oscillation.	0.0126	m s^{-1}
v_c	Casting velocity.	0.0045	m s^{-1}
ω	Frequency of the mould oscillation.	$4\pi/3$	rad s^{-1}
g	Constant due to gravity.	9.8	m s^{-2}

Table 3.1: Numerical values from [2].

Using the values in table 3.1, it can be obtained

$$[t] = 1.5s \tag{3.13}$$

and the parameters shown in table 3.2

To solve equation (3.4), an initial condition must be prescribed. In this case, it is suitable to set

$$\theta = 1. \tag{3.14}$$

ε	6.0193×10^{-5}.
$\mathcal{V}_0$	2.79
Γ	20.705
Λ	7.778
β	5.2045

Table 3.2: Some nondimensionalized parameters.

at $\tau = 0$ due to a cooling process starting at the mould wall.

Likewise, the region of an OM denoted by $\mathcal{R}$ is defined by

$$\mathcal{R} = \left\{ (X, Z) : -\frac{1}{2\pi} \cos^{-1}\left(\frac{1}{\mathcal{V}_0}\right) \leq Z \leq \frac{1}{2\pi} \cos^{-1}\left(\frac{1}{\mathcal{V}_0}\right),\, 0 \leq X \leq H(Z, \tau) \right\}. \tag{3.15}$$

and contains initially liquid flux. To solve equation (3.4) the solver Openfoam [5, 6] is employed, which is distributed with some tools to perform numerical computations using the finite-volume method as explained in [7]. In order, Openfoam solves the partial differential equations that models existing process in the field of science and engineering.

Hence, to solve equation (3.4) in the region $\mathcal{R}$, it is necessary to build a mesh of the region $\mathcal{R}$ suitable to employ the finite-volume method; in this sense, an utility called **blockMesh** included in the Openfoam software allows to build the mesh acoording to specific commands included in a file called **blockMeshDict**.

In figure 3.1 is shown the mesh of the region $\mathcal{R}$ built by the blockMesh utility and hence it can be observed that this region is the union of quadrilateral elements what makes suitable to employ the finite-volume method [7]; Thus the region $\mathcal{R}$ is decomposed into N quadrilaterals $\mathcal{R}_i$ for $i = 1, \ldots, N$, i.e.

$$\mathcal{R} = \bigcup_{i=1}^{N} \mathcal{R}_i. \tag{3.16}$$

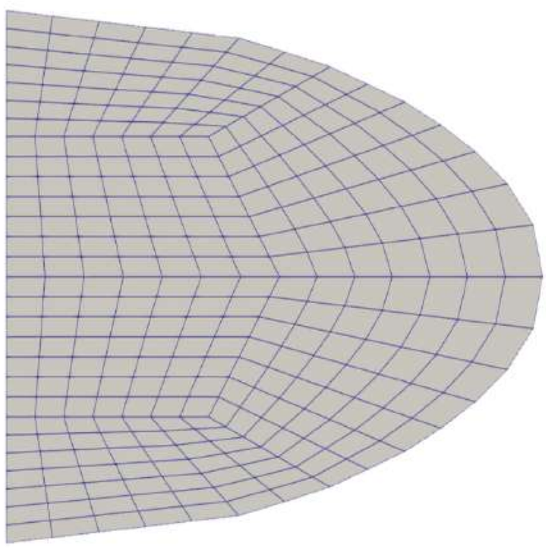

Figure 3.1: Mesh for the region of an OM built using the **blockMesh** utility.

Now, for every region $\mathcal{R}_i$, let $\mathcal{A}_i$ be the corresponding area, hence the average for the temperature θ in $\mathcal{R}_i$ is defined by

$$\bar{\theta}_i = \frac{1}{\mathcal{A}_i} \int_{\mathcal{R}_i} \theta(X, Z) d\mathcal{R}. \tag{3.17}$$

Also, by integrating equation (3.4) in the region $\mathcal{R}_i$ gives

$$\frac{\partial}{\partial \tau} \int_{\mathcal{R}_i} \theta \, d\mathcal{R} + \int_{\mathcal{R}_i} U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Z} d\mathcal{R} = \int_{\mathcal{R}_i} \frac{\partial^2 \theta}{\partial X^2} + \varepsilon \frac{\partial^2 \theta}{\partial Z^2} d\mathcal{R}, \tag{3.18}$$

and using (3.17) for the first term in equation (3.18) becomes

$$A_i \frac{\partial \bar{\theta}_i}{\partial \tau} + \int_{\mathcal{R}_i} \left(U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Z} \right) \theta d\mathcal{R} = \int_{\mathcal{R}_i} \Delta_\varepsilon \theta d\mathcal{R}, \tag{3.19}$$

where Δ_ε is a diffusion operator defined by

$$\Delta_\varepsilon := \frac{\partial^2}{\partial X^2} + \varepsilon \frac{\partial^2}{\partial Z^2}. \tag{3.20}$$

In equation (3.20) the second term in left hand side is the convection of the fluid flow and Δ_ε the diffusion for the liquid flux respectively. Hence equation (3.20) includes three terms: time variation of θ, convection and diffusion.

Openfoam provides a set of solvers [9] to obtain the numerical solution of wide range of partial differential equations; in this case, to solve equation (3.4), solver **scalarTransportFoam** is suitable to employ because it considers the terms previously mentioned. This solver expresses equation (3.4) as

$$fvm :: ddt(\theta) + fvm :: div(\phi, \theta) - fvm :: laplacian(DT, \theta), \tag{3.21}$$

where the first term is the time variation, the second term involves the convection and **laplacian** term is related to the diffusion term. Also, DT points out the thermal diffusivity in equation (3.21).

```
/* Solving Conv-Diff eq */
#include<fvCFD.H>
int main(){
  #include "createMesh.H"
  #include "createFields.H"
  Info<< "Solving T" << endl;
  solve(
        fvm::ddt(T) +
        fvm::div(phi,T) -
        fvm::laplacian(DT,T)
        );
  Info << "End." << endl;
}
```

Figure 3.2: Source code for the convection-diffusion equation in Openfoam software.

In figure 3.2 is shown the source code in the software Openfoam and which is written in the programming lenguage $C++$ for equation (3.21) and some lines of code indicate the construction of the mesh for one OM and the command **solve** allows the execution of the expression (3.21).

4. Results. The solver Openfoam solved equation (3.4) numerically by setting the $\theta-$values in the whole region $\mathcal{R}$ as $\theta = 1$ at $\tau = 0$. Subsequently, the temperature of the flux decreases due to the cooling effect at the mould wall $X = 0$ until $\theta = \theta_f$ where θ_f is the numerical value given by

$$\theta_f = \frac{T_f - T_w}{T_s - T_w} = 0.7284, \tag{4.1}$$

where $T_f = 1373K$ is the temperature of the phase change of the flux.

In the numerical computations the phase-transition of flux was reached at $\tau = \tau_f = 0.0025$ when $\theta = \theta_f$; from this, the lowest temperature is found at the mould wall and the highest temperature at the boundary $X = H(Z, \tau)$ where $\theta = 1$. Thus, to perform computations must be carried out up to τ_f when the onset of the solidification of the flux emerge and hence the formation of an OM is defined.

In figure 4.1 is shown the $\theta-$values obtained by the Openfoam software in three values of τ: $\tau = 0.0001$ (left), $\tau = 0.0010$ (center) and $\tau = 0.0025$ (right); where it is shown the decreasing of the $\theta-$values. Thus, at $\tau = 0.0025$ the onset of the solidification of flux emerges, according the $\theta-$values bar shown. Likewise, the arrows show fluid flow direction of the flux in time in these $\tau-$values.

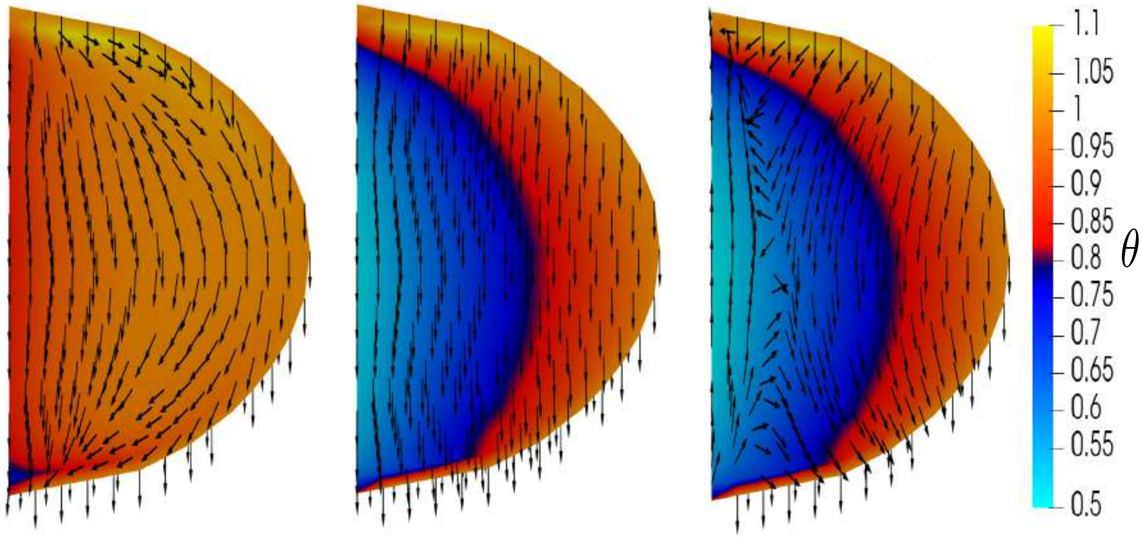

Figure 4.1: $\theta-$values in the region of an OM at three different values of τ: $\tau = 0.0001$ (left), $\tau = 0.0010$ (center) and $\tau = 0.0025$ (right).

5. Conclusiones. At the beginning, the liquid flux at $T = T_s$ temperature by the cooling effect at the mould wall becomes solid flux at $T = T_f$ and this way an OM is defined, this decreasing in the temperature was formulated in convection-diffusion equation in (2.6) that allowed to determine the temperature of the flux and hence the phase-transition from liquid to solid.

Using the Openfoam software which is based on the volume-finite method this temperature decreasing was possible to determine. Likewise, this shows the capability of Openfoam to solve partial differential equations related to science and engineering via three stages: **pre-process** where the mesh was built, **process** where the **solve** command was execute and **post-process** that allowed the visualization of the results as shown in figure 4.1.

In this work, the fluid flow for the flux is assumed to be in region $\mathcal{R}$ and initially the temperature $\theta = 1$ or $T = T_s$ at the boundary conditions at $X = 0$ decreases towards a rapid solidification of the flux as shown in figure 4.1 that points out a constant value for the steel temperature at $X = H(Z, \tau)$.

A future investigation can be focused on the solidification of the flux using three layers in the formation of an OM: solid flux, liquid flux and solid steel; in this case, the theory of the moving boundary problems must be considered and subsequently the recent results of the Stefan problem [8] can be suitable to employ.

Author contributions. The article has been developed interely for the author MZ.

Conflicts of interest. The author declare no conflict of interest.

Acknowledgment. I express my thanks to Dra Nelida Eladia Vicente Vicente by her support during the period of investigation of the present paper. Likewise, i would like to mention that this work had financial supported by the Universidad Nacional de Barranca via R.C.O No 843-2023-UNAB.

ORCID and License
Marcos Zambrano Fernández https://orcid.org/0000-0002-4327-9000

References

[1] Bland DR. Flux and the Continuous Casting of Steel. IMA Journal of Applied Mathematics. 89-112. 1984.
[2] Vynnycky M. and Zambrano M. Towards a moving point formulation for the modelling of oscillation mark formation in the continuous casting of steel. Appl. Math Model. Vol 63. 89-112. 1984.
[3] Vynnycky M, Saleem S, Devine KM, Florio BJ, Mitchell SL and OBrien SGB. On the formation of fold-type oscillation marks in the continuous casting of steel. Royal Society Open Science. Vol 4. 2017.
[4] Dantzig J. and Tucker C. Modeling in Material Processing. Cambridge University Press, 2012.
[5] Weller HG, Tabor G, Jasak H and Fureby C. A tensorial approach to computational continuum mechanics using object-oriented techniques. Computers in Physics. Vol 12. 1998.

[6] Chen G, Xion Q, Morris P, Paterson E, Sergeev A. and Wang Y. OpenFOAM for Computational Fluid Dynamics. AMS, Vol 61(4): 354-363. 2014.
[7] Leveque R. Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, 2002.
[8] Crank J. Free and Moving Boundary Problems. Oxford University Press. 3rd Edition. 1975.
[9] Elsevier Science, OpenFOAM - Field Operation and Manipulation, OpenFOAM foundation, 3rd Edition, 2012.