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Dolph algebras and Dolph groups

Álgebras y grupos de Dolph

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Abstract
A finite Hopf crossed product whose base ring is a finite field will be called a Dolph algebra, and the corresponding group of units will be called a Dolph group. Assuming known the crossed product of a ring and a group under a crossed mapping [1], the units of the cross products $\frac{\mathbb{Z}_2[X]}{\langle X^2\rangle} \rtimes_f C_2$ and $\frac{\mathbb{Z}_3[X]}{\langle X^2\rangle} \rtimes_f C_2$ are calculated. Furthermore, we give concrete examples of 4 classes of Hopf crossed products.
Keywords . Finite groups, crossed products, 2-cocycle, group of units, Dolph algebras.

Resumen
A un producto cruzado de Hopf finito cuyo anillo base es un cuerpo finito se le llamará álgebra de Dolph,y al correspondiente grupo de unidades, grupo de Dolph. Asumiendo conocido el producto cruzado de un anillo y un grupo bajo una aplicación cruzada [1], se calculan las unidades de los productos cruzados $\frac{\mathbb{Z}_2[X]}{\langle X^2\rangle} \rtimes_f C_2$ y $\frac{\mathbb{Z}_3[X]}{\langle X^2\rangle} \rtimes_f C_2$. Además, damos ejemplos concretos de 4 clases de productos cruzados de Hopf.
Palabras clave. Grupos finitos, productos cruzados, 2-cociclo, grupo de unidades, álgebras de Dolph.

1. Introducción. A la cuaterna (A, G, σ, f) se le llama un sistema cruzado si A es un álgebra, G es un grupo, $\sigma : G \to Aut(A)$ y $f : G \times G \to A^*$ son respectivamente, una función de G al grupo de automorfismos del álgebra A y una función de $G \times G$ al grupo A^* de las unidades de A, tales que para cualesquiera $g, g_0, g_1, g_2 \in G$ y $a \in A$ se satisfacen las siguientes condiciones:

1) $f(1_G, g) = f(g, 1_G) = 1_A$,
2) $f(g_1, g_2)^{g_0} f(g_0, g_1g_2) = f(g_0, g_1)f(g_0g_1, g_2)$,
3) $(a^{g_1})^{g_0} f(g_0, g_1) = f(g_0, g_1)a^{g_0g_1}$,

donde $\sigma(g)(a)$ se denota por a^g. La función σ se llama acción débil de G sobre A y la función f se llama $2-$cociclo.

Las condiciones $1)$ y $2)$ de la definición anterior son la condición de normalidad y condición de cociclo para la acción.

Sea (A, G, σ, f) un sistema cruzado. El álgebra (asociativa y unitaria) $\bigoplus_{g\in G} A\omega_g$ con multiplicación dada por las siguientes reglas:

$$\omega_g a = \sigma(g)(a)\omega_g \text{ y } \omega_g\omega_h = f(g, h)\omega_{gh} \text{ para } a \in A \text{ y } g, h \in G,$$

se llama producto cruzado determinado por σ y f, y denotaremos por $A \rtimes_f G$ [2].

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En el artículo [3], se calcularon algunos $2-$cociclos no triviales para productos cruzados de las extensiones algebraicas del cuerpo de los números racionales $\mathbb{Q}$ con grupos cíclicos de orden 2 y 3. En la cuarta sección del artículo, se obtuvo que $f = f_a$ es un $2-$cociclo no trivial para un producto cruzado $\frac{\mathbb{Q}[X]}{\langle P\rangle} \rtimes_f C_2$ cuando $a \in \mathbb{Q}^* \setminus \{1\}$ y P es un polinomio mónico.

Este artículo se inspira en obras de arte y combina las siguientes ideas: "Un escultor le dice a un niño: En esta piedra hay una hermosa obra de arte, solo hace falta sacar lo que sobra", y "Había un artista plástico que utilizaba el seudónimo Dolph, quién expresaba sus percepciones a través de cuadros abstractos. En algunas ocasiones, abordaba un tema en diferentes niveles mediante el uso de dos o más cuadros abstractos de pintura, es decir, creando un políptico".

El propósito de este trabajo es introducir el concepto de grupo de Dolph (Definición 3.2), caracterizado como el grupo de unidades de un producto cruzado de Hopf finito. Esto se logra al considerar los cuerpos finitos $\mathbb{Z}_2$ y $\mathbb{Z}_3$ en lugar del cuerpo de los números racionales $\mathbb{Q}$, representando los grupos mediante una tabla o múltiples tablas, y utilizando símbolos para simplificar la representación de sus elementos.

El cálculo de los grupos de Dolph sirve para mostrar que productos cruzados de Hopf tienen una aplicación en álgebra, proporcionando, por ejemplo, grupos finitos.

Introduciendo el concepto de discriminante para elementos del álgebra de Dolph, se logra describir el grupo de Dolph como aquel que consiste de aquellos elementos con discriminate diferente de cero. Esto guarda similitud con el grupo lineal $GL_2(\mathbb{R})$ que consiste de matrices cuadradas reales de orden 2×2 con determinante diferente de cero. En el contexto de geometría diferencial, el grupo especial lineal $SL_2(\mathbb{R})$ es un grupo de Lie que consiste de matrices cuadradas reales de orden 2×2 con determinante igual a 1.

Dado que el discriminante preserva los productos y unitarios, se deduce que los elementos del grupo de Dolph con discriminante 1 forman un grupo. En particular, $\overline{J}_3^*$ es un grupo de unidades (Proposición 3.1) para $J_3 = \frac{\mathbb{Z}_3[X]}{\langle X^2\rangle} \rtimes_f C_2$, donde la acción es dada por $g \cdot 1 = 1$, $g \cdot x = -x$; y el valor α del $2-$cociclo f es 2. Modificando solamente en el producto cruzado J_3 el cuerpo $\mathbb{Z}_3$ por el cuerpo de números reales $\mathbb{R}$, queda para un futuro trabajo de investigación determinar que el grupo de unidades el producto cruzado modificado con discriminante 1 es también un grupo de Lie, lo que mostraría que productos cruzados de Hopf tienen también aplicaciones en Geometría.

El presente trabajo está organizado de la siguiente manera:

En la segunda sección, dado un cuerpo K, se calculan los valores de un $2-$cociclo f para un producto cruzado de la forma $\frac{K[X]}{\langle X^2\rangle} \rtimes_f C_2$.

En la tercera sección, dado un cuerpo finito K, se calculan algunos grupos de unidades del producto cruzado $\frac{K[X]}{\langle X^2\rangle} \rtimes_f C_2$, donde $C_2 = \{1, g\}$ es un grupo cíclico de orden 2 y la acción de C_2 sobre $\frac{K[X]}{\langle X^2\rangle}$ es dada por $g \cdot 1 = 1$, $g \cdot x = -x$. Se consideran los casos en que K es $\mathbb{Z}_2$ y $\mathbb{Z}_3$. Además, se considera el caso en que la acción de C_2 sobre $\frac{\mathbb{Z}_3[X]}{\langle X^2\rangle}$ es dada por $g \cdot 1 = 1$, $g \cdot x = x$.

Dado que K es un cuerpo, $K[X]$ es un dominio euclidiano. Entonces para abreviar la expresión "clase residual de polinomios módulo un monomio de grado 2"se introduce la palabra clarpolis; de modo que $\frac{K[X]}{\langle X^2\rangle}$ es una K-álgebra de clarpolis.

2. Imagen de un 2-Cociclo en $\left(\frac{K[X]}{\langle X^2\rangle}\right)^*$.

Dado un producto cruzado de Hopf de la forma $E = \frac{K[X]}{\langle X^2\rangle} \rtimes_f C_2$ cuando K es un cuerpo y la acción del grupo cíclico C_2 de orden 2 sobre el álgebra de polinomios $K[X]$ cocientado por el ideal generado por X^2 es conocida, entonces se puede determinar el $2-$cociclo f de E.

Definición 2.1. [4, 2] *Sea K un cuerpo, A una $K-$álgebra y (A, G, σ, f) un sistema cruzado. Se llama producto cruzado de A con G a la $K-$álgebra $A \rtimes_f G$, que se define como el A-módulo libre $\bigoplus_{g\in G} A\omega_g$ provisto de la multiplicación dada por*

$$(a\omega_g)(b\omega_h) = ab^g f(g, h)\omega_{gh} \,. \tag{2.1}$$

El elemento identidad de $A \rtimes_f G$ es $1_A\omega_{1_G}$, en lo que sigue se escribe como ω_1.

Sea $E = A \rtimes_f C_2 = \dfrac{K[X]}{\langle X^2\rangle} \rtimes_f C_2$. Puesto que $A = \{a_0 + a_1 x \mid a_0, a_1 \in K\}$, $\{1, x\}$ es una base de A sobre K. Entonces se puede definir $C_2 = \{1, g\}$ como el grupo de automorfismos de A sobre K, dado por la tabla

	1	g
1	1	1
x	x	$-x$

donde $x = X + \langle X^2\rangle$, $g(1) = 1$, $g(x) = -x$.

Los elementos de A^* son conocidos. En efecto, si $a+bx \in A^*$, entonces existe $c+dx \in A$ tal que $(a+bx)(c+dx) = 1$. Usando la multiplicación se escribe $ac + (ad+bc)x + bdx^2 = 1$. Pero $x^2 = 0$, entonces $ac + (ad+bc)x = 1$. Esto ocurre si $ac = 1$ y $ad + bc = 0$. Así $c = a^{-1}$ y $d = -a^{-1}bc = -a^{-2}b$. Esto significa que $a + bx \in A^*$ si $a \neq 0$ y $b \in K$.
Por lo tanto, $A^* = \{a + bx \mid a \in K^* \text{ and } b \in K\}$. Además, se ve que $(a+bx)^{-1} = a^{-1} - a^{-2}bx$.

Ejemplo 2.1. *Sea $f : C_2 \times C_2 \to A^*$ un $2-$cociclo no trivial de $A \rtimes_f C_2$ tal que*

$$f(g^i, g^j) = \begin{cases} 1 & si\ i+j < 2 \\ \alpha & si\ i+j \geq 2\ , \end{cases} \tag{2.2}$$

donde $0 \leq i, j < 2$; $g^i, g^j \in C_2 = \{1, g\} = \langle g \rangle$. Si $Car(K) \neq 2$, entonces el valor de $\alpha = a \in K^*$.

Se considera $\sigma : C_2 \to Aut(A)$, donde $\sigma(g^i) : A \to A$ está definida por

$$\sigma(g^i)(a+bx) = (a+bx)^{g^i}.$$

Recordando que $C_2 \times C_2 = \{(1,1), (1,g), (g,1), (g,g)\}$. Si $(g^i, g^j) \in C_2 \times C_2$, se tiene $\sigma(g^i g^j) = \sigma(g^i)\sigma(g^j)$.

Esto significa que las igualdades siguientes son satisfechas
$\sigma(11) = \sigma(1)\sigma(1)$, $\sigma(1g) = \sigma(1)\sigma(g)$, $\sigma(g1) = \sigma(g)\sigma(1)$ y $\sigma(gg) = \sigma(g)\sigma(g)$.

En efecto, si $a + bx \in A = \dfrac{K[X]}{\langle X^2 \rangle}$, entonces :
$\sigma(11)(a+bx) = \sigma(1)(a+bx) = (a+bx)^1 = a+bx,$
$\sigma(1)\sigma(1)(a+bx) = \sigma(1)(a+bx)^1 = (a+bx)^1 = a+bx;$
$\sigma(1g)(a+bx) = \sigma(g)(a+bx) = (a+bx)^g = a-bx,$
$\sigma(1)\sigma(g)(a+bx) = \sigma(1)(a+bx)^g = (a-bx)^1 = a-bx;$
$\sigma(g1)(a+bx) = \sigma(g)(a+bx) = (a+bx)^g = a-bx,$
$\sigma(g)\sigma(1)(a+bx) = \sigma(g)(a+bx)^1 = (a+bx)^g = a-bx;$
$\sigma(gg)(a+bx) = \sigma(g^2)(a+bx) = \sigma(1)(a+bx) = a+bx,$
$\sigma(g)\sigma(g)(a+bx) = \sigma(g)(a+bx)^g = (a-bx)^g = a+bx.$
Claramente, $\sigma(1) = id_A$. La inversa de $\sigma(g)$ es $[\sigma(g)]^{-1} = \sigma(g)$ ya que $g^2 = 1$.
La aplicación $\sigma(g)$ es un automorfismo de la $K-$álgebra A.

Si $\lambda \in K$, $a + bx$ y $c + dx \in A$, entonces :

$$\begin{aligned} &1) \quad \sigma(g)(\lambda(a+bx)) = \lambda\sigma(g)(a+bx), \\ &2) \quad \sigma(g)((a+bx)+(c+dx)) = \sigma(g)(a+bx) + \sigma(g)(c+dx), \\ &3) \quad \sigma(g)((a+bx)(c+dx)) = \sigma(g)(a+bx)\sigma(g)(c+dx). \end{aligned}$$

En efecto :

$$\begin{aligned} 1)\ \sigma(g)(\lambda(a+bx)) &= \sigma(g)(\lambda a + \lambda bx) \\ &= (\lambda a + \lambda bx)^g \\ &= \lambda a - \lambda bx = \lambda(a-bx) = \lambda\sigma(g)(a+bx)\ ; \end{aligned}$$

$$\begin{aligned} 2)\ \sigma(g)((a+bx)+(c+dx)) &= \sigma(g)((a+c)+(b+d)x) \\ &= ((a+c)+(b+d)x)^g \\ &= (a+c)-(b+d)x = (a-bx)+(d-dx) \\ &= \sigma(g)(a+bx) + \sigma(g)(c+dx)\ ; \end{aligned}$$

$$\begin{aligned} 3)\ \sigma(g)((a+bx)(c+dx)) &= \sigma(g)(ac+(bc+ad)x) \\ &= (ac+(bc+ad)x)^g \\ &= ac-(bc+ad)x = (a-bx)(c-dx) \\ &= \sigma(g)(a+bx)\sigma(g)(c+dx). \end{aligned}$$

Por consiguiente, el homomorfismo de grupos $\sigma : C_2 \to Aut(A)$, $g^i \mapsto \sigma(g^i) : A \to A$ definido por $\sigma(g^i)(a+bx) = (a+bx)^{g^i}$ para $0 \leq i \leq 1$, es una acción de C_2 sobre A.

Por otro lado, dado que $\alpha \in A^*$, se sabe que $\alpha = a + bx$ donde $a \neq 0$. Según [5, Corollary 4.6], las tres condiciones siguientes deben ser satisfechas:

1) $f(1, g^i) = f(g^i, 1) = 1$,

2) $f(g^j, g^k)^{g^i} f(g^i, g^{j+k}) = f(g^i, g^j) f(g^{i+j}, g^k)$,

3) $((c+dx)^{g^j})^{g^i} f(g^i, g^j) = f(g^i, g^j)(c+dx)^{g^{i+j}}$, $(c+dx) \in A$.

$$\begin{aligned} & C_2 \times C_2 \times C_2 = \{1, g\} \times \{1, g\} \times \{1, g\} \\ = \ & \{(1,1,1),(1,1,g),(1,g,1),(1,g,g),(g,1,1),(g,1,g),(g,g,1),(g,g,g)\}. \end{aligned} \tag{2.3}$$

La afirmación 1) para f es satisfecha por definición.

La afirmación 3) para f, $((c+dx)^{g^j})^{g^i} f(g^i, g^j) = f(g^i, g^j)(c+dx)^{g^{i+j}}$, es satisfecha debido a que la $K-$álgebra $A = \dfrac{K[X]}{\langle X^2\rangle}$ es conmutativa y $C_2 \times A \to A$, $(g, a+bx) \mapsto (a+bx)^g = a - bx$ es una acción de C_2 sobre A.
Además la afirmación 2) para f, $f(g^j, g^k)^{g^i} f(g^i, g^{j+k}) = f(g^i, g^j) f(g^{i+j}, g^k)$, se satisface si, considerando (2.3), se cumplen las 8 igualdades siguientes:

(1) $1 = f(1,1)^1 f(1,(1)(1)) = f(1,1) f((1)(1),1)$,

(2) $1 = f(1,g)^1 f(1,(1)(g)) = f(1,1) f((1)(1),g)$,

(3) $1 = f(g,1)^1 f(1,(g)(1)) = f(1,g) f((1)(g),1)$,

(4) $\alpha = f(g,g)^1 f(1,(g)(g)) = f(1,g) f((1)(g),g)$,

(5) $1 = f(1,1)^g f(g,(1)(1)) = f(g,1) f((g)(1),1)$,

(6) $\alpha = f(1,g)^g f(g,(1)(g)) = f(g,1) f((g)(1),g)$,

(7) $\alpha = f(g,1)^g f(g,(g)(1)) = f(g,g) f((g)(g),1)$,

(8) $\alpha^g = f(g,g)^g f(g,g^2) = f(g,g) f(g^2,g) = \alpha$,

donde $\alpha^g = a - bx$ y $\alpha = a + bx$.

Puesto que Car$(K) \neq 2$, de la igualdad (8) se deduce que $\alpha = a$ ya que $b = 0$. Recordando el hecho de que $\alpha \in A^*$ se ve que $a \neq 0$. Por lo tanto $\alpha = a \in K^*$.

Observación 2.1. *Si Car$(K) = 2$, entonces la aplicación f definida en (2.2) es un $2-$cociclo de $A \rtimes_f C_2$ para todo $\alpha \in A^*$.*

Observación 2.2. *Si Car$(K) \neq 2$ y la acción de C_2 sobre A es dada por $g \cdot 1 = 1$, $g \cdot x = x$, entonces la aplicación f definida en (2.2) es un $2-$cociclo de $A \rtimes_f C_2$ para todo $\alpha \in A^*$.*

3. Los Grupos de Unidades de Productos Cruzados. El problema de hallar todas las unidades de un álgebra puede ser muy complicado. En el caso del álgebra $\frac{\mathbb{Z}_2[X]}{\langle X^2\rangle} \rtimes_f C_2$, la idea consiste en eliminar todos los elementos que son divisores de cero (sacar lo que sobra), para ello se construye la tabla de multiplicación de este álgebra debido a que es finita. De esta tabla se extrae una subtabla, que corresponde al grupo de unidades de dicha álgebra. Así, se ha obtenido dos de los tres grupos conmutativos de orden 8.

Replicando el procedimiento anterior para el álgebra $\frac{\mathbb{Z}_3[X]}{\langle X^2\rangle} \rtimes_f C_2$ se obtienen dos de los diez grupos no conmutativos de orden 36 [6] y dos de los cuatro grupos conmutativos de orden 36.
Por ejemplo, utilizando las tablas de multiplicación de E_2 y F_2 dadas en la página 5 y la páginas 6, respectivamente, vemos que:

1) $\rho_2\rho_5 = (x\omega_g)(x\omega_1) = x^2\omega_g = 0$. De este hecho sabemos que ρ_2 y ρ_5 son elementos divisores de cero de E_2.
2) $\rho_7\rho_8 = (x\omega_1 + \omega_g)(x\omega_1 + (1+x)\omega_g) = \rho_{13}$. Luego ρ_7 y ρ_8 son elementos de E_2, pero no son inversos el uno del otro.
3) $\eta_2\eta_5 = (x\omega_g)(x\omega_1) = x^2\omega_g = 0$. De este hecho sabemos que η_2 y η_5 son elementos divisores de cero de F_2.
4) $\eta_7\eta_8 = (x\omega_1 + \omega_g)(x\omega_1 + (1+x)\omega_g) = \omega_1 = \eta_9$. Luego η_7 y η_8 son unidades de F_2, y son inversos el uno del otro ya que $\eta_8\eta_7 = \eta_9$.

Utilizando como herramienta la estructura de álgebra llamada producto cruzado de Hopf [5, D 4.1], se ha identificado en total los 6 grupos finitos de ordenes 8 y 36 referidos anteriormente. Esto se ve en las tablas elaboradas de los grupos.

3.1. Productos Cruzados de la forma $\frac{\mathbb{Z}_2[X]}{\langle X^2\rangle} \rtimes_f C_2$. Notemos que $\dfrac{\mathbb{Z}_2[X]}{\langle X^2\rangle} = \{a + bx \mid a, b \in \mathbb{Z}_2\}$ donde $x = X + \langle X^2\rangle$. Puesto que $\mathbb{Z}_2 = \{0, 1\}$, $\dfrac{\mathbb{Z}_2[X]}{\langle X^2\rangle} = \{0, x, 1, 1+x\}$.

La adición y multiplicación de este anillo cociente están dadas por las siguientes tablas, respectivamente :

$+$	0	x	1	$1+x$
0	0	x	1	$1+x$
x	x	0	$1+x$	1
1	1	$1+x$	0	x
$1+x$	$1+x$	1	x	0

$*$	0	x	1	$1+x$
0	0	0	0	0
x	0	0	x	x
1	0	x	1	$1+x$
$1+x$	0	x	$1+x$	1

Si $A = \dfrac{\mathbb{Z}_2[X]}{\langle X^2 \rangle}$, entonces usando la tabla de multiplicación anterior $A^* = \{1, 1+x\}$.
Sea $C_2 = \{1, g\}$ un grupo cíclico de orden 2. Se define la acción de C_2 sobre A por $g \cdot 1 = 1$ y $g \cdot x = -x$. Utilizando la tabla de adición anterior, $g \cdot x = x$.

Para cada $\alpha \in A^*$, según la Observación 2.1, la aplicación $f_\alpha : C_2 \times C_2 \to A^*$ dada por

$$f_\alpha(g^i, g^j) = \begin{cases} 1 & \text{si } i+j < 2 \\ \alpha & \text{si } i+j \geq 2 \,, \end{cases}$$

donde $0 \leq i, j < 2$, es un 2-cociclo para el producto cruzado $A \rtimes_{f_\alpha} C_2$.

Para $\alpha = 1$, se denota $A \rtimes_{f_\alpha} C_2$ por E_2; para $\alpha = 1+x$, se denota $A \rtimes_{f_\alpha} C_2$ por F_2.

Se ve que $A \rtimes_{f_\alpha} C_2 = A\omega_1 \bigoplus A\omega_g = \{0, x\omega_g, \omega_g, (1+x)\omega_g, x\omega_1, x\omega_1 + x\omega_g, x\omega_1 + \omega_g,$ $x\omega_1 + (1+x)\omega_g, \omega_1, \omega_1 + x\omega_g, \omega_1 + \omega_g, \omega_1 + (1+x)\omega_g, (1+x)\omega_1, (1+x)\omega_1 + x\omega_g,$ $(1+x)\omega_1 + \omega_g, (1+x)\omega_1 + (1+x)\omega_g\}$.

La multiplicación en E_2 es dada por $a\omega_{g^i} b\omega_{g^j} = ab^{g^i} f(g^i, g^j)\omega_{g^{i+j}} = ab\omega_{g^{i+j}}$.

Se renombran los elementos de
$E_2 = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \rho_9, \rho_{10}, \rho_{11}, \rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{16}\}$ para construir su tabla de multiplicación.

$*$	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}
ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1
ρ_2	ρ_1	ρ_1	ρ_5	ρ_5	ρ_1	ρ_1	ρ_5	ρ_5	ρ_2	ρ_2	ρ_6	ρ_6	ρ_2	ρ_2	ρ_6	ρ_6
ρ_3	ρ_1	ρ_5	ρ_9	ρ_{13}	ρ_2	ρ_6	ρ_{10}	ρ_{14}	ρ_3	ρ_7	ρ_{11}	ρ_{15}	ρ_4	ρ_8	ρ_{12}	ρ_{16}
ρ_4	ρ_1	ρ_5	ρ_{13}	ρ_9	ρ_2	ρ_6	ρ_{14}	ρ_{10}	ρ_4	ρ_8	ρ_{16}	ρ_{12}	ρ_3	ρ_7	ρ_{15}	ρ_{11}
ρ_5	ρ_1	ρ_1	ρ_2	ρ_2	ρ_1	ρ_1	ρ_2	ρ_2	ρ_5	ρ_5	ρ_6	ρ_6	ρ_5	ρ_5	ρ_6	ρ_6
ρ_6	ρ_1	ρ_1	ρ_6	ρ_6	ρ_1	ρ_1	ρ_6	ρ_6	ρ_6	ρ_6	ρ_1	ρ_1	ρ_6	ρ_6	ρ_1	ρ_1
ρ_7	ρ_1	ρ_5	ρ_{10}	ρ_{14}	ρ_2	ρ_6	ρ_9	ρ_{13}	ρ_7	ρ_3	ρ_{16}	ρ_{12}	ρ_8	ρ_4	ρ_{15}	ρ_{11}
ρ_8	ρ_1	ρ_5	ρ_{14}	ρ_{10}	ρ_4	ρ_6	ρ_{13}	ρ_9	ρ_8	ρ_4	ρ_{11}	ρ_{15}	ρ_7	ρ_3	ρ_{12}	ρ_{16}
ρ_9	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}
ρ_{10}	ρ_1	ρ_2	ρ_7	ρ_8	ρ_5	ρ_6	ρ_3	ρ_4	ρ_{10}	ρ_9	ρ_{16}	ρ_{15}	ρ_{14}	ρ_{13}	ρ_{12}	ρ_{11}
ρ_{11}	ρ_1	ρ_6	ρ_{11}	ρ_{16}	ρ_6	ρ_1	ρ_{16}	ρ_{11}	ρ_{11}	ρ_{16}	ρ_1	ρ_6	ρ_{16}	ρ_{11}	ρ_6	ρ_1
ρ_{12}	ρ_1	ρ_6	ρ_{15}	ρ_{12}	ρ_6	ρ_1	ρ_{12}	ρ_{15}	ρ_{12}	ρ_{15}	ρ_6	ρ_1	ρ_{15}	ρ_{12}	ρ_1	ρ_6
ρ_{13}	ρ_1	ρ_2	ρ_4	ρ_3	ρ_5	ρ_6	ρ_8	ρ_7	ρ_{13}	ρ_{14}	ρ_{16}	ρ_{15}	ρ_9	ρ_{10}	ρ_{12}	ρ_{11}
ρ_{14}	ρ_1	ρ_2	ρ_8	ρ_7	ρ_5	ρ_6	ρ_4	ρ_3	ρ_{14}	ρ_{13}	ρ_{11}	ρ_{12}	ρ_{10}	ρ_9	ρ_{15}	ρ_{16}
ρ_{15}	ρ_1	ρ_6	ρ_{12}	ρ_{15}	ρ_6	ρ_1	ρ_{15}	ρ_{12}	ρ_{15}	ρ_{12}	ρ_6	ρ_1	ρ_{12}	ρ_{15}	ρ_1	ρ_6
ρ_{16}	ρ_1	ρ_6	ρ_{16}	ρ_{11}	ρ_6	ρ_1	ρ_{11}	ρ_{16}	ρ_{16}	ρ_{11}	ρ_1	ρ_6	ρ_{11}	ρ_{16}	ρ_6	ρ_1

El grupo multiplicativo de las unidades de E_2 es de orden 8 y está dado por la tabla siguiente:

*	ρ_3	ρ_4	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{13}	ρ_{14}
ρ_3	ρ_9	ρ_{13}	ρ_{10}	ρ_{14}	ρ_3	ρ_7	ρ_4	ρ_8
ρ_4	ρ_{13}	ρ_9	ρ_{14}	ρ_{10}	ρ_4	ρ_8	ρ_3	ρ_7
ρ_7	ρ_{10}	ρ_{14}	ρ_9	ρ_{13}	ρ_7	ρ_3	ρ_8	ρ_4
ρ_8	ρ_{14}	ρ_{10}	ρ_{13}	ρ_9	ρ_8	ρ_4	ρ_7	ρ_3
ρ_9	ρ_3	ρ_4	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{13}	ρ_{14}
ρ_{10}	ρ_7	ρ_8	ρ_3	ρ_4	ρ_{10}	ρ_9	ρ_{14}	ρ_{13}
ρ_{13}	ρ_4	ρ_3	ρ_8	ρ_7	ρ_{13}	ρ_{14}	ρ_9	ρ_{10}
ρ_{14}	ρ_8	ρ_7	ρ_4	ρ_3	ρ_{14}	ρ_{13}	ρ_{10}	ρ_9

La multiplicación en F_2 es dada por

$$a\omega_{g^i} b\omega_{g^j} = \begin{cases} ab\omega_{g^{i+j}} & \text{si } i+j<2 \\ ab(1+x)\omega_1 & \text{si } i+j=2 \,, \end{cases} \text{, donde } 0 \leq i,j < 2.$$

Se renombran los elementos de $F_2 = \{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8, \eta_9, \eta_{10}, \eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}, \eta_{15}, \eta_{16}\}$ para construir su tabla de multiplicación.

*	η_1	η_2	η_3	η_4	η_5	η_6	η_7	η_8	η_9	η_{10}	η_{11}	η_{12}	η_{13}	η_{14}	η_{15}	η_{16}
η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1	η_1
η_2	η_1	η_1	η_5	η_5	η_1	η_1	η_5	η_5	η_2	η_2	η_6	η_6	η_2	η_2	η_6	η_6
η_3	η_1	η_5	η_{13}	η_9	η_2	η_6	η_{14}	η_{10}	η_3	η_7	η_{15}	η_{11}	η_4	η_8	η_{16}	η_{12}
η_4	η_1	η_5	η_9	η_{13}	η_2	η_6	η_{10}	η_{14}	η_4	η_8	η_{12}	η_{16}	η_3	η_7	η_{11}	η_{15}
η_5	η_1	η_1	η_2	η_2	η_1	η_1	η_2	η_2	η_5	η_5	η_6	η_6	η_5	η_5	η_6	η_6
η_6	η_1	η_1	η_6	η_6	η_1	η_1	η_6	η_6	η_6	η_6	η_1	η_1	η_6	η_6	η_1	η_1
η_7	η_1	η_5	η_{14}	η_{10}	η_2	η_6	η_{13}	η_9	η_7	η_3	η_{12}	η_{16}	η_8	η_4	η_{11}	η_{15}
η_8	η_1	η_5	η_{10}	η_{14}	η_2	η_6	η_9	η_{13}	η_8	η_4	η_{15}	η_{11}	η_7	η_3	η_{16}	η_{12}
η_9	η_1	η_2	η_3	η_4	η_5	η_6	η_7	η_8	η_9	η_{10}	η_{11}	η_{12}	η_{13}	η_{14}	η_{15}	η_{16}
η_{10}	η_1	η_2	η_7	η_8	η_5	η_6	η_3	η_4	η_{10}	η_9	η_{16}	η_{15}	η_{14}	η_{13}	η_{12}	η_{11}
η_{11}	η_1	η_6	η_{15}	η_{12}	η_6	η_1	η_{12}	η_{15}	η_{11}	η_{16}	η_5	η_2	η_{16}	η_{11}	η_2	η_5
η_{12}	η_1	η_6	η_{11}	η_{16}	η_6	η_1	η_{16}	η_{11}	η_{12}	η_{15}	η_2	η_5	η_{15}	η_{12}	η_5	η_2
η_{13}	η_1	η_2	η_4	η_3	η_5	η_6	η_8	η_7	η_{13}	η_{14}	η_{16}	η_{15}	η_9	η_{10}	η_{12}	η_{11}
η_{14}	η_1	η_2	η_8	η_7	η_5	η_6	η_4	η_3	η_{14}	η_{13}	η_{11}	η_{12}	η_{10}	η_9	η_{15}	η_{16}
η_{15}	η_1	η_6	η_{16}	η_{11}	η_6	η_1	η_{11}	η_{16}	η_{15}	η_{12}	η_2	η_5	η_{12}	η_{15}	η_5	η_2
η_{16}	η_1	η_6	η_{12}	η_{15}	η_6	η_1	η_{15}	η_{12}	η_{16}	η_{11}	η_5	η_2	η_{11}	η_{16}	η_2	η_5

El grupo multiplicativo de las unidades de F_2 es de orden 8 y está dado por la tabla siguiente:

*	η_3	η_4	η_7	η_8	η_9	η_{10}	η_{13}	η_{14}
η_3	η_{13}	η_9	η_{14}	η_{10}	η_3	η_7	η_4	η_8
η_4	η_9	η_{13}	η_{10}	η_{14}	η_4	η_8	η_3	η_7
η_7	η_{14}	η_{10}	η_{13}	η_9	η_7	η_3	η_8	η_4
η_8	η_{10}	η_{14}	η_9	η_{13}	η_8	η_4	η_7	η_3
η_9	η_3	η_4	η_7	η_8	η_9	η_{10}	η_{13}	η_{14}
η_{10}	η_7	η_8	η_3	η_4	η_{10}	η_9	η_{14}	η_{13}
η_{13}	η_4	η_3	η_8	η_7	η_{13}	η_{14}	η_9	η_{10}
η_{14}	η_8	η_7	η_4	η_3	η_{14}	η_{13}	η_{10}	η_9

3.2. Productos Cruzados de la forma $\frac{\mathbb{Z}_3[X]}{\langle X^2\rangle} \rtimes_f C_2$. Como en el caso anterior, se obtiene:
$\dfrac{\mathbb{Z}_3[X]}{\langle X^2\rangle} = \{0, x, 2x, 1, 1+x, 1+2x, 2, 2+x, 2+2x\}$.

La adición y multiplicación de este anillo cociente están dadas por las siguientes tablas, respectivamente:

+	0	x	$2x$	1	$1+x$	$1+2x$	2	$2+x$	$2+2x$
0	0	x	$2x$	1	$1+x$	$1+2x$	2	$2+x$	$2+2x$
x	x	$2x$	0	$1+x$	$1+2x$	1	$2+x$	$2+2x$	2
$2x$	$2x$	0	x	$1+2x$	1	$1+x$	$2+2x$	2	$2+x$
1	1	$1+x$	$1+2x$	2	$2+x$	$2+2x$	0	x	$2x$
$1+x$	$1+x$	$1+2x$	1	$2+x$	$2+2x$	2	x	$2x$	0
$1+2x$	$1+2x$	1	$1+x$	$2+2x$	2	$2+x$	$2x$	0	x
2	2	$2+x$	$2+2x$	0	x	$2x$	1	$1+x$	$1+2x$
$2+x$	$2+x$	$2+2x$	2	x	$2x$	0	$1+x$	$1+2x$	1
$2+2x$	$2+2x$	2	$2+x$	$2x$	0	x	$1+2x$	1	$1+x$

*	0	x	$2x$	1	$1+x$	$1+2x$	2	$2+x$	$2+2x$
0	0	0	0	0	0	0	0	0	0
x	0	0	0	x	x	x	$2x$	$2x$	$2x$
$2x$	0	0	0	$2x$	$2x$	$2x$	x	x	x
1	0	x	$2x$	1	$1+x$	$1+2x$	2	$2+x$	$2+2x$
$1+x$	0	x	$2x$	$1+x$	$1+2x$	1	$2+2x$	2	$2+x$
$1+2x$	0	x	$2x$	$1+2x$	1	$1+x$	$2+x$	$2+2x$	2
2	0	$2x$	x	2	$2+2x$	$2+x$	1	$1+2x$	$1+x$
$2+x$	0	$2x$	x	$2+x$	2	$2+2x$	$1+2x$	$1+x$	1
$2+2x$	0	$2x$	x	$2+2x$	$2+x$	2	$1+x$	1	$1+2x$

Sea $A = \dfrac{\mathbb{Z}_3[X]}{\langle X^2\rangle}$. Si la acción de C_2 sobre A es dada por $g\cdot 1 = 1$, $g\cdot x = -x$, al producto cruzado $A \rtimes_{f_\alpha} C_2$, denotaremos por I_3 cuando $\alpha = 1$; por J_3 cuando $\alpha = 2$. En el caso en que $g\cdot 1 = 1$, $g \cdot x = x$, dicho producto cruzado lo denotaremos por E_3 cuando $\alpha = 1$; por F_3 cuando $\alpha = 2$.

Utilizando la tabla de multiplicación de $\dfrac{\mathbb{Z}_3[X]}{\langle X^2\rangle}$ y la fórmula (2.1), calcularemos los productos de elementos de I_3, J_3, E_3 y F_3. Por ejemplo, veremos algunos productos de elementos de I_3 y J_3, en las páginas 9 y 15, respectivamente:

1) $\kappa_{27}\kappa_{29} = [2x\omega_1 + (2+2x)\omega_g]\,[\omega_1 + x\omega_g] = \kappa_9$,
2) $\kappa_{29}\kappa_{27} = [\omega_1 + x\omega_g]\,[2x\omega_1 + (2+2x)\omega_g] = \kappa_{18}$,
3) $\upsilon_6\upsilon_{75} = [(1+2x)\omega_g]\,[(2+2x)\omega_1 + 2x\omega_g] = \upsilon_{27}$,
4) $\upsilon_{75}\upsilon_6 = [(2+2x)\omega_1 + 2x\omega_g]\,[1+2x)\omega_g] = \upsilon_{16}$.

Si $A = \dfrac{\mathbb{Z}_3[X]}{\langle X^2\rangle}$, entonces observando la tabla de multiplicación de A vemos que $A^* = \{1, 2, 1+x, 2+x, 1+2x, 2+2x\}$. Sea $C_2 = \{1, g\}$ un grupo cíclico de orden 2. Se define la acción de C_2 sobre A por $g\cdot 1 = 1$ y $g\cdot x = -x$. Los únicos elementos α de A^* tales que $g\cdot\alpha = \alpha$ son 1 y 2.

Para cada $\alpha \in A^*$ tal que $g\cdot\alpha = \alpha$, por el Ejemplo 2.1, la aplicación $f_\alpha : C_2 \times C_2 \to A^*$ definida por

$$f_\alpha(g^i, g^j) = \begin{cases} 1 & \text{si } i+j < 2 \\ \alpha & \text{si } i+j \geq 2\,, \end{cases}$$

donde $0 \leq i, j < 2$, es un 2-cociclo para el producto cruzado $A \rtimes_{f_\alpha} C_2$.

Para $\alpha = 1$, se denota $A \rtimes_{f_\alpha} C_2$ por I_3; para $\alpha = 2$, se denota $A \rtimes_{f_\alpha} C_2$ por J_3.

$A \rtimes_{f_\alpha} C_2 = A\omega_1 \bigoplus A\omega_g = \{0, x\omega_g, 2x\omega_g, \omega_g, (1+x)\omega_g, (1+2x)\omega_g, 2\omega_g, (2+x)\omega_g, (2+2x)\omega_g, x\omega_1, x\omega_1 + x\omega_g, x\omega_1 + 2x\omega_g, x\omega_1 + \omega_g, x\omega_1 + (1+x)\omega_g, x\omega_1 + (1+2x)\omega_g, x\omega_1 + 2\omega_g, x\omega_1 + (2+x)\omega_g, x\omega_1 + (2+2x)\omega_g, 2x\omega_1, 2x\omega_1 + x\omega_g, 2x\omega_1 + 2x\omega_g, 2x\omega_1 + \omega_g, 2x\omega_1 + (1+x)\omega_g, 2x\omega_1 + (1+2x)\omega_g, 2x\omega_1 +$

$2\omega_g, 2x\omega_1+(2+x)\omega_g, 2x\omega_1+(2+2x)\omega_g, \omega_1, \omega_1+x\omega_g, \omega_1+2x\omega_g, \omega_1+\omega_g, \omega_1+(1+x)\omega_g, \omega_1+(1+2x)\omega_g, \omega_1+2\omega_g, \omega_1+(2+x)\omega_g, \omega_1+(2+2x)\omega_g, (1+x)\omega_1, (1+x)\omega_1+x\omega_g, (1+x)\omega_1+2x\omega_g, (1+x)\omega_1+\omega_g, (1+x)\omega_1+(1+x)\omega_g, (1+x)\omega_1+(1+2x)\omega_g, (1+x)\omega_1+2\omega_g, (1+x)\omega_1+(2+x)\omega_g, (1+x)\omega_1+(2+2x)\omega_g, (1+2x)\omega_1, (1+2x)\omega_1+x\omega_g, (1+2x)\omega_1+2x\omega_g, (1+2x)\omega_1+\omega_g, (1+2x)\omega_1+(1+x)\omega_g, (1+2x)\omega_1+(1+2x)\omega_g, (1+2x)\omega_1+2\omega_g, (1+2x)\omega_1+(2+x)\omega_g, (1+2x)\omega_1+(2+2x)\omega_g, 2\omega_1, 2\omega_1+x\omega_g, 2\omega_1+2x\omega_g, 2\omega_1+\omega_g, 2\omega_1+(1+x)\omega_g, 2\omega_1+(1+2x)\omega_g, 2\omega_1+2\omega_g, 2\omega_1+(2+x)\omega_g, 2\omega_1+(2+2x)\omega_g, (2+x)\omega_1, (2+x)\omega_1+x\omega_g, (2+x)\omega_1+2x\omega_g, (2+x)\omega_1+\omega_g, (2+x)\omega_1+(1+x)\omega_g, (2+x)\omega_1+(1+2x)\omega_g, (2+x)\omega_1+2\omega_g, (2+x)\omega_1+(2+x)\omega_g, (2+x)\omega_1+(2+2x)\omega_g, (2+2x)\omega_1, (2+2x)\omega_1+x\omega_g, (2+2x)\omega_1+2x\omega_g, (2+2x)\omega_1+\omega_g, (2+2x)\omega_1+(1+x)\omega_g, (2+2x)\omega_1+(1+2x)\omega_g, (2+2x)\omega_1+2\omega_g, (2+2x)\omega_1+(2+x)\omega_g, (2+2x)\omega_1+(2+2x)\omega_g\}$.

La multiplicación en I_3 es dada por $a\omega_{g^i} b\omega_{g^j} = ab^{g^i}\omega_{g^{i+j}}$.

Se renombran los elementos de I_3 preservando el mismo orden de la siguiente manera:
$I_3 = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6, \kappa_7, \kappa_8, \kappa_9, \kappa_{10}, \kappa_{11}, \kappa_{12}, \kappa_{13}, \kappa_{14}, \kappa_{15}, \kappa_{16}, \kappa_{17}, \kappa_{18}, \kappa_{19}, \kappa_{20}, \kappa_{21}, \kappa_{22}, \kappa_{23}, \kappa_{24}, \kappa_{25}, \kappa_{26}, \kappa_{27}, \kappa_{28}, \kappa_{29}, \kappa_{30}, \kappa_{31}, \kappa_{32}, \kappa_{33}, \kappa_{34}, \kappa_{35}, \kappa_{36}, \kappa_{37}, \kappa_{38}, \kappa_{39}, \kappa_{40}, \kappa_{41}, \kappa_{42}, \kappa_{43}, \kappa_{44}, \kappa_{45}, \kappa_{46}, \kappa_{47}, \kappa_{48}, \kappa_{49}, \kappa_{50}, \kappa_{51}, \kappa_{52}, \kappa_{53}, \kappa_{54}, \kappa_{55}, \kappa_{56}, \kappa_{57}, \kappa_{58}, \kappa_{59}, \kappa_{60}, \kappa_{61}, \kappa_{62}, \kappa_{63}, \kappa_{64}, \kappa_{65}, \kappa_{66}, \kappa_{67}, \kappa_{68}, \kappa_{69}, \kappa_{70}, \kappa_{71}, \kappa_{72}, \kappa_{73}, \kappa_{74}, \kappa_{75}, \kappa_{76}, \kappa_{77}, \kappa_{78}, \kappa_{79}, \kappa_{80}, \kappa_{81}\}$, con el propósito de construir su tabla de multiplicación. Dicha tabla consiste de cinco tablas más pequeñas, cada una de ellas es distribuida en una página, como se muestra a continuación:

*	κ_{1}	κ_{2}	κ_{3}	κ_{4}	κ_{5}	κ_{6}	κ_{7}	κ_{8}	κ_{9}	κ_{10}	κ_{11}	κ_{12}	κ_{13}	κ_{14}	κ_{15}	κ_{16}
κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}
κ_{2}	κ_{1}	κ_{1}	κ_{1}	κ_{10}	κ_{10}	κ_{10}	κ_{19}	κ_{19}	κ_{19}	κ_{1}	κ_{1}	κ_{1}	κ_{10}	κ_{10}	κ_{10}	κ_{19}
κ_{3}	κ_{1}	κ_{1}	κ_{1}	κ_{19}	κ_{19}	κ_{19}	κ_{10}	κ_{10}	κ_{10}	κ_{1}	κ_{1}	κ_{1}	κ_{19}	κ_{19}	κ_{19}	κ_{10}
κ_{4}	κ_{1}	κ_{19}	κ_{10}	κ_{28}	κ_{46}	κ_{37}	κ_{55}	κ_{73}	κ_{64}	κ_{3}	κ_{21}	κ_{12}	κ_{30}	κ_{48}	κ_{39}	κ_{57}
κ_{5}	κ_{1}	κ_{19}	κ_{10}	κ_{37}	κ_{28}	κ_{46}	κ_{73}	κ_{64}	κ_{55}	κ_{3}	κ_{21}	κ_{12}	κ_{39}	κ_{30}	κ_{48}	κ_{75}
κ_{6}	κ_{1}	κ_{19}	κ_{10}	κ_{46}	κ_{37}	κ_{28}	κ_{64}	κ_{55}	κ_{73}	κ_{3}	κ_{21}	κ_{12}	κ_{48}	κ_{39}	κ_{30}	κ_{66}
κ_{7}	κ_{1}	κ_{10}	κ_{19}	κ_{55}	κ_{64}	κ_{73}	κ_{28}	κ_{37}	κ_{46}	κ_{2}	κ_{11}	κ_{20}	κ_{56}	κ_{65}	κ_{74}	κ_{29}
κ_{8}	κ_{1}	κ_{10}	κ_{19}	κ_{64}	κ_{73}	κ_{55}	κ_{46}	κ_{28}	κ_{37}	κ_{2}	κ_{11}	κ_{20}	κ_{65}	κ_{74}	κ_{56}	κ_{47}
κ_{9}	κ_{1}	κ_{10}	κ_{19}	κ_{73}	κ_{55}	κ_{64}	κ_{37}	κ_{46}	κ_{28}	κ_{2}	κ_{11}	κ_{20}	κ_{74}	κ_{56}	κ_{65}	κ_{38}
κ_{10}	κ_{1}	κ_{1}	κ_{1}	κ_{2}	κ_{2}	κ_{2}	κ_{3}	κ_{3}	κ_{3}	κ_{1}	κ_{1}	κ_{1}	κ_{2}	κ_{2}	κ_{2}	κ_{3}
κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{21}	κ_{21}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{21}
κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{12}
κ_{13}	κ_{1}	κ_{19}	κ_{10}	κ_{29}	κ_{47}	κ_{38}	κ_{57}	κ_{75}	κ_{66}	κ_{3}	κ_{21}	κ_{12}	κ_{28}	κ_{46}	κ_{37}	κ_{56}
κ_{14}	κ_{1}	κ_{19}	κ_{10}	κ_{38}	κ_{29}	κ_{47}	κ_{75}	κ_{66}	κ_{57}	κ_{3}	κ_{21}	κ_{12}	κ_{37}	κ_{28}	κ_{46}	κ_{74}
κ_{15}	κ_{1}	κ_{19}	κ_{10}	κ_{47}	κ_{38}	κ_{29}	κ_{66}	κ_{57}	κ_{75}	κ_{3}	κ_{21}	κ_{12}	κ_{46}	κ_{37}	κ_{28}	κ_{65}
κ_{16}	κ_{1}	κ_{10}	κ_{19}	κ_{56}	κ_{65}	κ_{74}	κ_{30}	κ_{39}	κ_{48}	κ_{2}	κ_{11}	κ_{20}	κ_{57}	κ_{66}	κ_{75}	κ_{28}
κ_{17}	κ_{1}	κ_{10}	κ_{19}	κ_{65}	κ_{74}	κ_{56}	κ_{48}	κ_{30}	κ_{39}	κ_{2}	κ_{11}	κ_{20}	κ_{66}	κ_{75}	κ_{57}	κ_{46}
κ_{18}	κ_{1}	κ_{10}	κ_{19}	κ_{74}	κ_{56}	κ_{65}	κ_{39}	κ_{48}	κ_{30}	κ_{2}	κ_{11}	κ_{20}	κ_{75}	κ_{57}	κ_{66}	κ_{37}
κ_{19}	κ_{1}	κ_{1}	κ_{1}	κ_{3}	κ_{3}	κ_{3}	κ_{2}	κ_{2}	κ_{2}	κ_{1}	κ_{1}	κ_{1}	κ_{3}	κ_{3}	κ_{3}	κ_{2}
κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{20}
κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{11}	κ_{11}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{11}
κ_{22}	κ_{1}	κ_{19}	κ_{10}	κ_{30}	κ_{48}	κ_{39}	κ_{56}	κ_{74}	κ_{65}	κ_{3}	κ_{21}	κ_{12}	κ_{29}	κ_{47}	κ_{38}	κ_{55}
κ_{23}	κ_{1}	κ_{19}	κ_{10}	κ_{39}	κ_{30}	κ_{48}	κ_{74}	κ_{65}	κ_{56}	κ_{3}	κ_{21}	κ_{12}	κ_{38}	κ_{29}	κ_{47}	κ_{73}
κ_{24}	κ_{1}	κ_{19}	κ_{10}	κ_{48}	κ_{39}	κ_{30}	κ_{65}	κ_{56}	κ_{74}	κ_{3}	κ_{21}	κ_{12}	κ_{47}	κ_{38}	κ_{29}	κ_{64}
κ_{25}	κ_{1}	κ_{10}	κ_{19}	κ_{57}	κ_{66}	κ_{75}	κ_{29}	κ_{38}	κ_{47}	κ_{2}	κ_{11}	κ_{20}	κ_{55}	κ_{64}	κ_{73}	κ_{30}
κ_{26}	κ_{1}	κ_{10}	κ_{19}	κ_{66}	κ_{75}	κ_{57}	κ_{47}	κ_{29}	κ_{38}	κ_{2}	κ_{11}	κ_{20}	κ_{64}	κ_{73}	κ_{55}	κ_{48}
κ_{27}	κ_{1}	κ_{10}	κ_{19}	κ_{75}	κ_{57}	κ_{66}	κ_{38}	κ_{47}	κ_{29}	κ_{2}	κ_{11}	κ_{20}	κ_{73}	κ_{55}	κ_{64}	κ_{39}
κ_{28}	κ_{1}	κ_{2}	κ_{3}	κ_{4}	κ_{5}	κ_{6}	κ_{7}	κ_{8}	κ_{9}	κ_{10}	κ_{11}	κ_{12}	κ_{13}	κ_{14}	κ_{15}	κ_{16}
κ_{29}	κ_{1}	κ_{2}	κ_{3}	κ_{13}	κ_{14}	κ_{15}	κ_{25}	κ_{26}	κ_{27}	κ_{10}	κ_{11}	κ_{12}	κ_{22}	κ_{23}	κ_{24}	κ_{7}
κ_{30}	κ_{1}	κ_{2}	κ_{3}	κ_{22}	κ_{23}	κ_{24}	κ_{16}	κ_{17}	κ_{18}	κ_{10}	κ_{11}	κ_{12}	κ_{4}	κ_{5}	κ_{6}	κ_{25}
κ_{31}	κ_{1}	κ_{20}	κ_{12}	κ_{31}	κ_{50}	κ_{42}	κ_{61}	κ_{80}	κ_{72}	κ_{12}	κ_{1}	κ_{20}	κ_{42}	κ_{31}	κ_{50}	κ_{72}
κ_{32}	κ_{1}	κ_{20}	κ_{12}	κ_{40}	κ_{51}	κ_{40}	κ_{79}	κ_{71}	κ_{63}	κ_{12}	κ_{1}	κ_{20}	κ_{51}	κ_{40}	κ_{32}	κ_{63}
κ_{33}	κ_{1}	κ_{20}	κ_{12}	κ_{49}	κ_{41}	κ_{33}	κ_{70}	κ_{62}	κ_{81}	κ_{12}	κ_{1}	κ_{20}	κ_{33}	κ_{49}	κ_{41}	κ_{81}
κ_{34}	κ_{1}	κ_{11}	κ_{21}	κ_{58}	κ_{68}	κ_{78}	κ_{34}	κ_{44}	κ_{54}	κ_{11}	κ_{21}	κ_{1}	κ_{68}	κ_{78}	κ_{58}	κ_{44}
κ_{35}	κ_{1}	κ_{11}	κ_{21}	κ_{67}	κ_{77}	κ_{60}	κ_{52}	κ_{35}	κ_{45}	κ_{11}	κ_{21}	κ_{1}	κ_{77}	κ_{60}	κ_{67}	κ_{35}
κ_{36}	κ_{1}	κ_{11}	κ_{21}	κ_{76}	κ_{59}	κ_{69}	κ_{43}	κ_{53}	κ_{36}	κ_{11}	κ_{21}	κ_{1}	κ_{59}	κ_{69}	κ_{76}	κ_{53}
κ_{37}	κ_{1}	κ_{2}	κ_{3}	κ_{5}	κ_{6}	κ_{4}	κ_{9}	κ_{7}	κ_{8}	κ_{10}	κ_{11}	κ_{12}	κ_{14}	κ_{15}	κ_{13}	κ_{18}
κ_{38}	κ_{1}	κ_{2}	κ_{3}	κ_{14}	κ_{15}	κ_{13}	κ_{27}	κ_{25}	κ_{26}	κ_{10}	κ_{11}	κ_{12}	κ_{23}	κ_{24}	κ_{22}	κ_{9}
κ_{39}	κ_{1}	κ_{2}	κ_{3}	κ_{23}	κ_{24}	κ_{22}	κ_{18}	κ_{16}	κ_{17}	κ_{10}	κ_{11}	κ_{12}	κ_{5}	κ_{6}	κ_{4}	κ_{27}
κ_{40}	κ_{1}	κ_{20}	κ_{12}	κ_{32}	κ_{51}	κ_{40}	κ_{63}	κ_{79}	κ_{71}	κ_{12}	κ_{1}	κ_{20}	κ_{40}	κ_{32}	κ_{51}	κ_{71}
κ_{41}	κ_{1}	κ_{20}	κ_{12}	κ_{41}	κ_{33}	κ_{49}	κ_{81}	κ_{70}	κ_{62}	κ_{12}	κ_{1}	κ_{20}	κ_{49}	κ_{41}	κ_{33}	κ_{62}
κ_{42}	κ_{1}	κ_{20}	κ_{12}	κ_{50}	κ_{42}	κ_{31}	κ_{72}	κ_{61}	κ_{80}	κ_{12}	κ_{1}	κ_{20}	κ_{31}	κ_{50}	κ_{42}	κ_{80}
κ_{43}	κ_{1}	κ_{11}	κ_{21}	κ_{59}	κ_{69}	κ_{76}	κ_{36}	κ_{43}	κ_{53}	κ_{11}	κ_{21}	κ_{1}	κ_{69}	κ_{76}	κ_{59}	κ_{43}
κ_{44}	κ_{1}	κ_{11}	κ_{21}	κ_{68}	κ_{78}	κ_{58}	κ_{54}	κ_{34}	κ_{44}	κ_{11}	κ_{21}	κ_{1}	κ_{78}	κ_{58}	κ_{68}	κ_{34}
κ_{45}	κ_{1}	κ_{11}	κ_{21}	κ_{77}	κ_{60}	κ_{67}	κ_{45}	κ_{52}	κ_{35}	κ_{11}	κ_{21}	κ_{1}	κ_{60}	κ_{67}	κ_{77}	κ_{52}
κ_{46}	κ_{1}	κ_{2}	κ_{3}	κ_{6}	κ_{4}	κ_{5}	κ_{8}	κ_{9}	κ_{7}	κ_{10}	κ_{11}	κ_{12}	κ_{15}	κ_{13}	κ_{14}	κ_{17}
κ_{47}	κ_{1}	κ_{2}	κ_{3}	κ_{15}	κ_{13}	κ_{14}	κ_{26}	κ_{27}	κ_{25}	κ_{10}	κ_{11}	κ_{12}	κ_{24}	κ_{22}	κ_{23}	κ_{8}
κ_{48}	κ_{1}	κ_{2}	κ_{3}	κ_{24}	κ_{22}	κ_{23}	κ_{17}	κ_{18}	κ_{16}	κ_{10}	κ_{11}	κ_{12}	κ_{6}	κ_{4}	κ_{5}	κ_{26}
κ_{49}	κ_{1}	κ_{20}	κ_{12}	κ_{33}	κ_{49}	κ_{41}	κ_{62}	κ_{81}	κ_{70}	κ_{12}	κ_{1}	κ_{20}	κ_{41}	κ_{33}	κ_{49}	κ_{70}
κ_{50}	κ_{1}	κ_{20}	κ_{12}	κ_{42}	κ_{31}	κ_{50}	κ_{80}	κ_{72}	κ_{61}	κ_{12}	κ_{1}	κ_{20}	κ_{50}	κ_{42}	κ_{31}	κ_{61}
κ_{51}	κ_{1}	κ_{20}	κ_{12}	κ_{51}	κ_{40}	κ_{32}	κ_{71}	κ_{63}	κ_{79}	κ_{12}	κ_{1}	κ_{20}	κ_{32}	κ_{51}	κ_{40}	κ_{79}
κ_{52}	κ_{1}	κ_{11}	κ_{21}	κ_{60}	κ_{67}	κ_{77}	κ_{35}	κ_{45}	κ_{52}	κ_{11}	κ_{21}	κ_{1}	κ_{67}	κ_{77}	κ_{60}	κ_{45}
κ_{53}	κ_{1}	κ_{11}	κ_{21}	κ_{69}	κ_{76}	κ_{59}	κ_{53}	κ_{36}	κ_{43}	κ_{11}	κ_{21}	κ_{1}	κ_{76}	κ_{59}	κ_{69}	κ_{36}
κ_{54}	κ_{1}	κ_{11}	κ_{21}	κ_{78}	κ_{58}	κ_{68}	κ_{44}	κ_{54}	κ_{34}	κ_{11}	κ_{21}	κ_{1}	κ_{58}	κ_{68}	κ_{78}	κ_{54}
κ_{55}	κ_{1}	κ_{3}	κ_{2}	κ_{7}	κ_{9}	κ_{8}	κ_{4}	κ_{6}	κ_{5}	κ_{19}	κ_{21}	κ_{20}	κ_{25}	κ_{27}	κ_{26}	κ_{22}
κ_{56}	κ_{1}	κ_{3}	κ_{2}	κ_{16}	κ_{18}	κ_{17}	κ_{22}	κ_{24}	κ_{23}	κ_{19}	κ_{21}	κ_{20}	κ_{7}	κ_{9}	κ_{8}	κ_{13}
κ_{57}	κ_{1}	κ_{3}	κ_{2}	κ_{25}	κ_{27}	κ_{26}	κ_{13}	κ_{15}	κ_{14}	κ_{19}	κ_{21}	κ_{22}	κ_{16}	κ_{18}	κ_{17}	κ_{4}
κ_{58}	κ_{1}	κ_{21}	κ_{11}	κ_{34}	κ_{54}	κ_{44}	κ_{58}	κ_{78}	κ_{68}	κ_{21}	κ_{11}	κ_{1}	κ_{54}	κ_{44}	κ_{34}	κ_{78}
κ_{59}	κ_{1}	κ_{21}	κ_{11}	κ_{43}	κ_{36}	κ_{53}	κ_{76}	κ_{69}	κ_{59}	κ_{21}	κ_{11}	κ_{1}	κ_{36}	κ_{53}	κ_{43}	κ_{69}
κ_{60}	κ_{1}	κ_{21}	κ_{11}	κ_{52}	κ_{45}	κ_{35}	κ_{67}	κ_{60}	κ_{77}	κ_{21}	κ_{11}	κ_{1}	κ_{45}	κ_{35}	κ_{52}	κ_{60}
κ_{61}	κ_{1}	κ_{12}	κ_{20}	κ_{61}	κ_{72}	κ_{80}	κ_{31}	κ_{42}	κ_{50}	κ_{20}	κ_{1}	κ_{12}	κ_{80}	κ_{61}	κ_{72}	κ_{50}
κ_{62}	κ_{1}	κ_{12}	κ_{20}	κ_{70}	κ_{81}	κ_{62}	κ_{49}	κ_{33}	κ_{41}	κ_{20}	κ_{1}	κ_{12}	κ_{62}	κ_{70}	κ_{81}	κ_{41}
κ_{63}	κ_{1}	κ_{12}	κ_{20}	κ_{79}	κ_{63}	κ_{71}	κ_{40}	κ_{51}	κ_{32}	κ_{20}	κ_{1}	κ_{12}	κ_{71}	κ_{79}	κ_{63}	κ_{32}
κ_{64}	κ_{1}	κ_{3}	κ_{2}	κ_{8}	κ_{7}	κ_{9}	κ_{6}	κ_{5}	κ_{4}	κ_{19}	κ_{21}	κ_{20}	κ_{26}	κ_{25}	κ_{27}	κ_{24}
κ_{65}	κ_{1}	κ_{3}	κ_{2}	κ_{17}	κ_{16}	κ_{18}	κ_{24}	κ_{23}	κ_{22}	κ_{19}	κ_{21}	κ_{20}	κ_{8}	κ_{7}	κ_{9}	κ_{15}
κ_{66}	κ_{1}	κ_{3}	κ_{2}	κ_{26}	κ_{25}	κ_{27}	κ_{15}	κ_{14}	κ_{13}	κ_{19}	κ_{21}	κ_{20}	κ_{17}	κ_{16}	κ_{18}	κ_{6}
κ_{67}	κ_{1}	κ_{21}	κ_{11}	κ_{35}	κ_{52}	κ_{45}	κ_{60}	κ_{77}	κ_{67}	κ_{21}	κ_{11}	κ_{1}	κ_{52}	κ_{45}	κ_{35}	κ_{77}
κ_{68}	κ_{1}	κ_{21}	κ_{11}	κ_{44}	κ_{34}	κ_{54}	κ_{78}	κ_{68}	κ_{58}	κ_{21}	κ_{11}	κ_{1}	κ_{34}	κ_{54}	κ_{44}	κ_{68}
κ_{69}	κ_{1}	κ_{21}	κ_{11}	κ_{53}	κ_{43}	κ_{36}	κ_{69}	κ_{59}	κ_{76}	κ_{21}	κ_{11}	κ_{1}	κ_{43}	κ_{36}	κ_{53}	κ_{59}
κ_{70}	κ_{1}	κ_{12}	κ_{20}	κ_{62}	κ_{70}	κ_{81}	κ_{33}	κ_{41}	κ_{49}	κ_{20}	κ_{1}	κ_{12}	κ_{81}	κ_{62}	κ_{70}	κ_{49}
κ_{71}	κ_{1}	κ_{12}	κ_{20}	κ_{71}	κ_{79}	κ_{63}	κ_{51}	κ_{32}	κ_{40}	κ_{20}	κ_{1}	κ_{12}	κ_{63}	κ_{71}	κ_{79}	κ_{40}
κ_{72}	κ_{1}	κ_{12}	κ_{20}	κ_{80}	κ_{61}	κ_{72}	κ_{42}	κ_{50}	κ_{31}	κ_{20}	κ_{1}	κ_{12}	κ_{72}	κ_{80}	κ_{61}	κ_{31}
κ_{73}	κ_{1}	κ_{3}	κ_{2}	κ_{9}	κ_{8}	κ_{7}	κ_{5}	κ_{4}	κ_{6}	κ_{19}	κ_{21}	κ_{20}	κ_{27}	κ_{26}	κ_{25}	κ_{23}
κ_{74}	κ_{1}	κ_{3}	κ_{2}	κ_{18}	κ_{17}	κ_{16}	κ_{23}	κ_{22}	κ_{24}	κ_{19}	κ_{21}	κ_{20}	κ_{9}	κ_{8}	κ_{7}	κ_{14}
κ_{75}	κ_{1}	κ_{3}	κ_{2}	κ_{27}	κ_{26}	κ_{25}	κ_{14}	κ_{13}	κ_{15}	κ_{19}	κ_{21}	κ_{20}	κ_{18}	κ_{17}	κ_{16}	κ_{5}
κ_{76}	κ_{1}	κ_{21}	κ_{11}	κ_{36}	κ_{53}	κ_{43}	κ_{59}	κ_{76}	κ_{69}	κ_{21}	κ_{11}	κ_{1}	κ_{53}	κ_{43}	κ_{36}	κ_{76}
κ_{77}	κ_{1}	κ_{21}	κ_{11}	κ_{45}	κ_{35}	κ_{52}	κ_{77}	κ_{67}	κ_{60}	κ_{21}	κ_{11}	κ_{1}	κ_{35}	κ_{52}	κ_{45}	κ_{67}
κ_{78}	κ_{1}	κ_{21}	κ_{11}	κ_{54}	κ_{44}	κ_{34}	κ_{68}	κ_{58}	κ_{78}	κ_{21}	κ_{11}	κ_{1}	κ_{44}	κ_{34}	κ_{54}	κ_{58}
κ_{79}	κ_{1}	κ_{12}	κ_{20}	κ_{63}	κ_{71}	κ_{79}	κ_{32}	κ_{40}	κ_{51}	κ_{20}	κ_{1}	κ_{12}	κ_{79}	κ_{63}	κ_{71}	κ_{51}
κ_{80}	κ_{1}	κ_{12}	κ_{20}	κ_{72}	κ_{80}	κ_{61}	κ_{50}	κ_{31}	κ_{42}	κ_{20}	κ_{1}	κ_{12}	κ_{61}	κ_{72}	κ_{80}	κ_{42}
κ_{81}	κ_{1}	κ_{12}	κ_{20}	κ_{81}	κ_{62}	κ_{70}	κ_{41}	κ_{49}	κ_{33}	κ_{20}	κ_{1}	κ_{12}	κ_{70}	κ_{81}	κ_{62}	κ_{33}

*	κ_{17}	κ_{18}	κ_{19}	κ_{20}	κ_{21}	κ_{22}	κ_{23}	κ_{24}	κ_{25}	κ_{26}	κ_{27}	κ_{28}	κ_{29}	κ_{30}	κ_{31}	κ_{32}
κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}
κ_{2}	κ_{19}	κ_{19}	κ_{1}	κ_{1}	κ_{1}	κ_{10}	κ_{10}	κ_{10}	κ_{19}	κ_{19}	κ_{19}	κ_{2}	κ_{2}	κ_{2}	κ_{11}	κ_{11}
κ_{3}	κ_{10}	κ_{10}	κ_{1}	κ_{1}	κ_{1}	κ_{19}	κ_{19}	κ_{19}	κ_{10}	κ_{10}	κ_{10}	κ_{3}	κ_{3}	κ_{3}	κ_{21}	κ_{21}
κ_{4}	κ_{75}	κ_{66}	κ_{2}	κ_{20}	κ_{11}	κ_{29}	κ_{47}	κ_{38}	κ_{56}	κ_{74}	κ_{65}	κ_{4}	κ_{22}	κ_{13}	κ_{31}	κ_{49}
κ_{5}	κ_{66}	κ_{57}	κ_{2}	κ_{20}	κ_{11}	κ_{38}	κ_{29}	κ_{47}	κ_{74}	κ_{65}	κ_{56}	κ_{5}	κ_{23}	κ_{14}	κ_{41}	κ_{32}
κ_{6}	κ_{57}	κ_{75}	κ_{2}	κ_{20}	κ_{11}	κ_{47}	κ_{38}	κ_{29}	κ_{65}	κ_{56}	κ_{74}	κ_{6}	κ_{24}	κ_{15}	κ_{51}	κ_{42}
κ_{7}	κ_{38}	κ_{47}	κ_{3}	κ_{12}	κ_{21}	κ_{57}	κ_{66}	κ_{75}	κ_{30}	κ_{39}	κ_{48}	κ_{7}	κ_{16}	κ_{25}	κ_{61}	κ_{70}
κ_{8}	κ_{29}	κ_{38}	κ_{3}	κ_{12}	κ_{21}	κ_{66}	κ_{75}	κ_{57}	κ_{48}	κ_{30}	κ_{39}	κ_{8}	κ_{17}	κ_{26}	κ_{71}	κ_{80}
κ_{9}	κ_{47}	κ_{29}	κ_{3}	κ_{12}	κ_{21}	κ_{75}	κ_{57}	κ_{66}	κ_{39}	κ_{48}	κ_{30}	κ_{9}	κ_{18}	κ_{27}	κ_{81}	κ_{63}
κ_{10}	κ_{3}	κ_{3}	κ_{1}	κ_{1}	κ_{1}	κ_{2}	κ_{2}	κ_{2}	κ_{3}	κ_{3}	κ_{3}	κ_{10}	κ_{10}	κ_{10}	κ_{11}	κ_{11}
κ_{11}	κ_{21}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{21}	κ_{21}	κ_{21}	κ_{11}	κ_{11}	κ_{11}	κ_{21}	κ_{21}
κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{12}	κ_{12}	κ_{12}	κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}
κ_{13}	κ_{74}	κ_{65}	κ_{2}	κ_{20}	κ_{11}	κ_{30}	κ_{48}	κ_{39}	κ_{55}	κ_{73}	κ_{64}	κ_{13}	κ_{4}	κ_{22}	κ_{41}	κ_{32}
κ_{14}	κ_{65}	κ_{56}	κ_{2}	κ_{20}	κ_{11}	κ_{39}	κ_{30}	κ_{48}	κ_{73}	κ_{64}	κ_{55}	κ_{14}	κ_{5}	κ_{23}	κ_{51}	κ_{42}
κ_{15}	κ_{56}	κ_{74}	κ_{2}	κ_{20}	κ_{11}	κ_{48}	κ_{39}	κ_{30}	κ_{64}	κ_{55}	κ_{73}	κ_{15}	κ_{6}	κ_{24}	κ_{31}	κ_{49}
κ_{16}	κ_{37}	κ_{46}	κ_{3}	κ_{12}	κ_{21}	κ_{55}	κ_{64}	κ_{73}	κ_{29}	κ_{38}	κ_{47}	κ_{16}	κ_{25}	κ_{7}	κ_{71}	κ_{80}
κ_{17}	κ_{28}	κ_{37}	κ_{3}	κ_{12}	κ_{21}	κ_{64}	κ_{73}	κ_{55}	κ_{47}	κ_{29}	κ_{38}	κ_{17}	κ_{26}	κ_{8}	κ_{81}	κ_{63}
κ_{18}	κ_{46}	κ_{28}	κ_{3}	κ_{12}	κ_{21}	κ_{73}	κ_{55}	κ_{64}	κ_{38}	κ_{47}	κ_{29}	κ_{18}	κ_{27}	κ_{9}	κ_{61}	κ_{70}
κ_{19}	κ_{2}	κ_{2}	κ_{1}	κ_{1}	κ_{1}	κ_{3}	κ_{3}	κ_{3}	κ_{2}	κ_{2}	κ_{2}	κ_{19}	κ_{19}	κ_{19}	κ_{21}	κ_{21}
κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{20}	κ_{20}	κ_{20}	κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}
κ_{21}	κ_{11}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{11}	κ_{11}	κ_{11}	κ_{21}	κ_{21}	κ_{21}	κ_{11}	κ_{11}
κ_{22}	κ_{73}	κ_{64}	κ_{2}	κ_{20}	κ_{11}	κ_{28}	κ_{46}	κ_{37}	κ_{57}	κ_{75}	κ_{66}	κ_{22}	κ_{13}	κ_{4}	κ_{51}	κ_{42}
κ_{23}	κ_{64}	κ_{55}	κ_{2}	κ_{20}	κ_{11}	κ_{37}	κ_{28}	κ_{46}	κ_{75}	κ_{66}	κ_{57}	κ_{23}	κ_{14}	κ_{5}	κ_{31}	κ_{49}
κ_{24}	κ_{55}	κ_{73}	κ_{2}	κ_{20}	κ_{11}	κ_{46}	κ_{37}	κ_{28}	κ_{66}	κ_{57}	κ_{75}	κ_{24}	κ_{15}	κ_{6}	κ_{41}	κ_{32}
κ_{25}	κ_{39}	κ_{48}	κ_{3}	κ_{12}	κ_{21}	κ_{56}	κ_{65}	κ_{74}	κ_{28}	κ_{37}	κ_{46}	κ_{25}	κ_{7}	κ_{16}	κ_{81}	κ_{63}
κ_{26}	κ_{30}	κ_{39}	κ_{3}	κ_{12}	κ_{21}	κ_{65}	κ_{74}	κ_{56}	κ_{46}	κ_{28}	κ_{37}	κ_{26}	κ_{8}	κ_{17}	κ_{61}	κ_{70}
κ_{27}	κ_{48}	κ_{30}	κ_{3}	κ_{12}	κ_{21}	κ_{74}	κ_{56}	κ_{65}	κ_{73}	κ_{55}	κ_{64}	κ_{27}	κ_{9}	κ_{18}	κ_{71}	κ_{80}
κ_{28}	κ_{17}	κ_{18}	κ_{19}	κ_{20}	κ_{21}	κ_{22}	κ_{23}	κ_{24}	κ_{25}	κ_{26}	κ_{27}	κ_{28}	κ_{29}	κ_{30}	κ_{31}	κ_{32}
κ_{29}	κ_{8}	κ_{9}	κ_{19}	κ_{20}	κ_{21}	κ_{4}	κ_{5}	κ_{6}	κ_{16}	κ_{17}	κ_{18}	κ_{29}	κ_{30}	κ_{28}	κ_{41}	κ_{42}
κ_{30}	κ_{26}	κ_{27}	κ_{19}	κ_{20}	κ_{21}	κ_{13}	κ_{14}	κ_{15}	κ_{7}	κ_{8}	κ_{9}	κ_{30}	κ_{28}	κ_{29}	κ_{51}	κ_{49}
κ_{31}	κ_{61}	κ_{80}	κ_{20}	κ_{12}	κ_{1}	κ_{50}	κ_{42}	κ_{31}	κ_{80}	κ_{72}	κ_{61}	κ_{31}	κ_{50}	κ_{42}	κ_{61}	κ_{80}
κ_{32}	κ_{79}	κ_{71}	κ_{20}	κ_{12}	κ_{1}	κ_{32}	κ_{51}	κ_{40}	κ_{71}	κ_{63}	κ_{79}	κ_{32}	κ_{51}	κ_{40}	κ_{71}	κ_{63}
κ_{33}	κ_{70}	κ_{62}	κ_{20}	κ_{12}	κ_{1}	κ_{41}	κ_{33}	κ_{49}	κ_{62}	κ_{81}	κ_{70}	κ_{33}	κ_{49}	κ_{41}	κ_{81}	κ_{70}
κ_{34}	κ_{54}	κ_{34}	κ_{21}	κ_{1}	κ_{11}	κ_{78}	κ_{58}	κ_{68}	κ_{54}	κ_{34}	κ_{44}	κ_{34}	κ_{44}	κ_{54}	κ_{1}	κ_{11}
κ_{35}	κ_{45}	κ_{52}	κ_{21}	κ_{1}	κ_{11}	κ_{60}	κ_{67}	κ_{77}	κ_{45}	κ_{52}	κ_{35}	κ_{35}	κ_{45}	κ_{52}	κ_{11}	κ_{21}
κ_{36}	κ_{36}	κ_{43}	κ_{21}	κ_{1}	κ_{11}	κ_{69}	κ_{76}	κ_{59}	κ_{53}	κ_{36}	κ_{43}	κ_{36}	κ_{43}	κ_{53}	κ_{21}	κ_{1}
κ_{37}	κ_{16}	κ_{17}	κ_{19}	κ_{20}	κ_{21}	κ_{23}	κ_{24}	κ_{22}	κ_{27}	κ_{25}	κ_{26}	κ_{37}	κ_{38}	κ_{39}	κ_{41}	κ_{42}
κ_{38}	κ_{7}	κ_{8}	κ_{19}	κ_{20}	κ_{21}	κ_{5}	κ_{6}	κ_{4}	κ_{18}	κ_{16}	κ_{17}	κ_{38}	κ_{39}	κ_{37}	κ_{51}	κ_{49}
κ_{39}	κ_{25}	κ_{26}	κ_{19}	κ_{20}	κ_{21}	κ_{14}	κ_{15}	κ_{13}	κ_{9}	κ_{7}	κ_{8}	κ_{39}	κ_{37}	κ_{38}	κ_{31}	κ_{32}
κ_{40}	κ_{63}	κ_{79}	κ_{20}	κ_{12}	κ_{1}	κ_{51}	κ_{40}	κ_{32}	κ_{79}	κ_{71}	κ_{63}	κ_{40}	κ_{32}	κ_{51}	κ_{71}	κ_{63}
κ_{41}	κ_{81}	κ_{70}	κ_{20}	κ_{12}	κ_{1}	κ_{33}	κ_{52}	κ_{44}	κ_{70}	κ_{62}	κ_{81}	κ_{41}	κ_{33}	κ_{49}	κ_{81}	κ_{70}
κ_{42}	κ_{72}	κ_{61}	κ_{20}	κ_{12}	κ_{1}	κ_{42}	κ_{31}	κ_{50}	κ_{61}	κ_{80}	κ_{72}	κ_{42}	κ_{31}	κ_{50}	κ_{61}	κ_{80}
κ_{43}	κ_{53}	κ_{36}	κ_{21}	κ_{1}	κ_{11}	κ_{76}	κ_{59}	κ_{69}	κ_{53}	κ_{36}	κ_{43}	κ_{43}	κ_{53}	κ_{36}	κ_{11}	κ_{21}
κ_{44}	κ_{44}	κ_{54}	κ_{21}	κ_{1}	κ_{11}	κ_{58}	κ_{68}	κ_{78}	κ_{44}	κ_{54}	κ_{34}	κ_{44}	κ_{54}	κ_{34}	κ_{21}	κ_{1}
κ_{45}	κ_{35}	κ_{45}	κ_{21}	κ_{1}	κ_{11}	κ_{67}	κ_{77}	κ_{60}	κ_{35}	κ_{45}	κ_{52}	κ_{45}	κ_{52}	κ_{35}	κ_{1}	κ_{11}
κ_{46}	κ_{18}	κ_{16}	κ_{19}	κ_{20}	κ_{21}	κ_{24}	κ_{22}	κ_{23}	κ_{26}	κ_{27}	κ_{25}	κ_{46}	κ_{47}	κ_{48}	κ_{51}	κ_{49}
κ_{47}	κ_{9}	κ_{7}	κ_{19}	κ_{20}	κ_{21}	κ_{6}	κ_{4}	κ_{5}	κ_{17}	κ_{18}	κ_{16}	κ_{47}	κ_{48}	κ_{46}	κ_{31}	κ_{32}
κ_{48}	κ_{27}	κ_{25}	κ_{19}	κ_{20}	κ_{21}	κ_{15}	κ_{13}	κ_{14}	κ_{8}	κ_{9}	κ_{7}	κ_{48}	κ_{46}	κ_{47}	κ_{41}	κ_{42}
κ_{49}	κ_{62}	κ_{81}	κ_{20}	κ_{12}	κ_{1}	κ_{49}	κ_{41}	κ_{33}	κ_{81}	κ_{70}	κ_{62}	κ_{49}	κ_{41}	κ_{33}	κ_{81}	κ_{70}
κ_{50}	κ_{80}	κ_{72}	κ_{20}	κ_{12}	κ_{1}	κ_{31}	κ_{50}	κ_{42}	κ_{72}	κ_{61}	κ_{80}	κ_{50}	κ_{42}	κ_{31}	κ_{61}	κ_{80}
κ_{51}	κ_{71}	κ_{63}	κ_{20}	κ_{12}	κ_{1}	κ_{40}	κ_{32}	κ_{51}	κ_{63}	κ_{79}	κ_{71}	κ_{51}	κ_{40}	κ_{32}	κ_{71}	κ_{63}
κ_{52}	κ_{52}	κ_{35}	κ_{21}	κ_{1}	κ_{11}	κ_{77}	κ_{60}	κ_{67}	κ_{52}	κ_{35}	κ_{45}	κ_{52}	κ_{35}	κ_{45}	κ_{21}	κ_{1}
κ_{53}	κ_{43}	κ_{53}	κ_{21}	κ_{1}	κ_{11}	κ_{59}	κ_{69}	κ_{76}	κ_{43}	κ_{53}	κ_{36}	κ_{53}	κ_{36}	κ_{43}	κ_{1}	κ_{11}
κ_{54}	κ_{58}	κ_{44}	κ_{21}	κ_{1}	κ_{11}	κ_{68}	κ_{78}	κ_{58}	κ_{34}	κ_{44}	κ_{54}	κ_{54}	κ_{34}	κ_{44}	κ_{11}	κ_{21}
κ_{55}	κ_{24}	κ_{23}	κ_{10}	κ_{12}	κ_{11}	κ_{16}	κ_{18}	κ_{17}	κ_{13}	κ_{15}	κ_{14}	κ_{55}	κ_{57}	κ_{56}	κ_{61}	κ_{63}
κ_{56}	κ_{15}	κ_{14}	κ_{10}	κ_{12}	κ_{11}	κ_{25}	κ_{27}	κ_{26}	κ_{4}	κ_{6}	κ_{5}	κ_{56}	κ_{55}	κ_{57}	κ_{71}	κ_{70}
κ_{57}	κ_{6}	κ_{5}	κ_{10}	κ_{12}	κ_{11}	κ_{7}	κ_{9}	κ_{8}	κ_{22}	κ_{24}	κ_{23}	κ_{57}	κ_{56}	κ_{55}	κ_{81}	κ_{80}
κ_{58}	κ_{68}	κ_{58}	κ_{11}	κ_{1}	κ_{21}	κ_{44}	κ_{34}	κ_{54}	κ_{68}	κ_{58}	κ_{78}	κ_{58}	κ_{78}	κ_{68}	κ_{1}	κ_{21}
κ_{59}	κ_{59}	κ_{76}	κ_{11}	κ_{1}	κ_{21}	κ_{53}	κ_{43}	κ_{36}	κ_{59}	κ_{76}	κ_{69}	κ_{59}	κ_{76}	κ_{69}	κ_{11}	κ_{1}
κ_{60}	κ_{77}	κ_{67}	κ_{11}	κ_{1}	κ_{21}	κ_{35}	κ_{52}	κ_{45}	κ_{77}	κ_{67}	κ_{60}	κ_{60}	κ_{77}	κ_{67}	κ_{21}	κ_{11}
κ_{61}	κ_{31}	κ_{42}	κ_{12}	κ_{20}	κ_{1}	κ_{72}	κ_{80}	κ_{61}	κ_{42}	κ_{50}	κ_{31}	κ_{61}	κ_{72}	κ_{80}	κ_{31}	κ_{42}
κ_{62}	κ_{49}	κ_{33}	κ_{12}	κ_{20}	κ_{1}	κ_{81}	κ_{62}	κ_{70}	κ_{33}	κ_{41}	κ_{49}	κ_{62}	κ_{70}	κ_{81}	κ_{41}	κ_{49}
κ_{63}	κ_{40}	κ_{51}	κ_{12}	κ_{20}	κ_{1}	κ_{63}	κ_{71}	κ_{79}	κ_{51}	κ_{32}	κ_{40}	κ_{63}	κ_{71}	κ_{79}	κ_{51}	κ_{32}
κ_{64}	κ_{23}	κ_{22}	κ_{10}	κ_{12}	κ_{11}	κ_{17}	κ_{16}	κ_{18}	κ_{15}	κ_{14}	κ_{13}	κ_{64}	κ_{66}	κ_{65}	κ_{71}	κ_{70}
κ_{65}	κ_{14}	κ_{13}	κ_{10}	κ_{12}	κ_{11}	κ_{26}	κ_{25}	κ_{27}	κ_{6}	κ_{5}	κ_{4}	κ_{65}	κ_{64}	κ_{66}	κ_{81}	κ_{80}
κ_{66}	κ_{5}	κ_{4}	κ_{10}	κ_{12}	κ_{11}	κ_{8}	κ_{7}	κ_{9}	κ_{24}	κ_{23}	κ_{22}	κ_{66}	κ_{65}	κ_{64}	κ_{61}	κ_{63}
κ_{67}	κ_{67}	κ_{60}	κ_{11}	κ_{1}	κ_{21}	κ_{45}	κ_{35}	κ_{52}	κ_{67}	κ_{60}	κ_{77}	κ_{67}	κ_{60}	κ_{77}	κ_{11}	κ_{1}
κ_{68}	κ_{58}	κ_{78}	κ_{11}	κ_{1}	κ_{21}	κ_{54}	κ_{44}	κ_{34}	κ_{58}	κ_{78}	κ_{68}	κ_{68}	κ_{58}	κ_{78}	κ_{21}	κ_{11}
κ_{69}	κ_{76}	κ_{69}	κ_{11}	κ_{1}	κ_{21}	κ_{36}	κ_{53}	κ_{43}	κ_{76}	κ_{69}	κ_{59}	κ_{69}	κ_{59}	κ_{76}	κ_{1}	κ_{21}
κ_{70}	κ_{33}	κ_{41}	κ_{12}	κ_{20}	κ_{1}	κ_{70}	κ_{81}	κ_{62}	κ_{41}	κ_{49}	κ_{33}	κ_{70}	κ_{81}	κ_{62}	κ_{41}	κ_{49}
κ_{71}	κ_{51}	κ_{32}	κ_{12}	κ_{20}	κ_{1}	κ_{79}	κ_{63}	κ_{71}	κ_{32}	κ_{40}	κ_{51}	κ_{71}	κ_{79}	κ_{63}	κ_{51}	κ_{32}
κ_{72}	κ_{42}	κ_{50}	κ_{12}	κ_{20}	κ_{1}	κ_{61}	κ_{72}	κ_{80}	κ_{50}	κ_{31}	κ_{42}	κ_{72}	κ_{80}	κ_{61}	κ_{31}	κ_{42}
κ_{73}	κ_{22}	κ_{24}	κ_{10}	κ_{12}	κ_{11}	κ_{18}	κ_{17}	κ_{16}	κ_{14}	κ_{13}	κ_{15}	κ_{73}	κ_{75}	κ_{74}	κ_{81}	κ_{80}
κ_{74}	κ_{13}	κ_{15}	κ_{10}	κ_{12}	κ_{11}	κ_{27}	κ_{26}	κ_{25}	κ_{5}	κ_{4}	κ_{6}	κ_{74}	κ_{73}	κ_{75}	κ_{61}	κ_{63}
κ_{75}	κ_{4}	κ_{6}	κ_{10}	κ_{12}	κ_{11}	κ_{9}	κ_{8}	κ_{7}	κ_{23}	κ_{22}	κ_{24}	κ_{75}	κ_{74}	κ_{73}	κ_{71}	κ_{70}
κ_{76}	κ_{69}	κ_{59}	κ_{11}	κ_{1}	κ_{21}	κ_{43}	κ_{36}	κ_{53}	κ_{69}	κ_{59}	κ_{76}	κ_{76}	κ_{69}	κ_{59}	κ_{21}	κ_{11}
κ_{77}	κ_{60}	κ_{77}	κ_{11}	κ_{1}	κ_{21}	κ_{52}	κ_{45}	κ_{35}	κ_{60}	κ_{77}	κ_{67}	κ_{77}	κ_{67}	κ_{60}	κ_{1}	κ_{21}
κ_{78}	κ_{78}	κ_{68}	κ_{11}	κ_{1}	κ_{21}	κ_{34}	κ_{54}	κ_{44}	κ_{78}	κ_{68}	κ_{58}	κ_{78}	κ_{68}	κ_{58}	κ_{11}	κ_{1}
κ_{79}	κ_{32}	κ_{40}	κ_{12}	κ_{20}	κ_{1}	κ_{71}	κ_{79}	κ_{63}	κ_{40}	κ_{51}	κ_{32}	κ_{79}	κ_{63}	κ_{71}	κ_{51}	κ_{32}
κ_{80}	κ_{50}	κ_{31}	κ_{12}	κ_{20}	κ_{1}	κ_{80}	κ_{61}	κ_{72}	κ_{31}	κ_{42}	κ_{50}	κ_{80}	κ_{61}	κ_{72}	κ_{31}	κ_{42}
κ_{81}	κ_{41}	κ_{49}	κ_{12}	κ_{20}	κ_{1}	κ_{62}	κ_{70}	κ_{81}	κ_{49}	κ_{33}	κ_{41}	κ_{81}	κ_{62}	κ_{70}	κ_{41}	κ_{49}

*	κ_{33}	κ_{34}	κ_{35}	κ_{36}	κ_{37}	κ_{38}	κ_{39}	κ_{40}	κ_{41}	κ_{42}	κ_{43}	κ_{44}	κ_{45}	κ_{46}	κ_{47}	κ_{48}
κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}
κ_{2}	κ_{11}	κ_{20}	κ_{20}	κ_{20}	κ_{2}	κ_{2}	κ_{2}	κ_{11}	κ_{11}	κ_{11}	κ_{20}	κ_{20}	κ_{20}	κ_{2}	κ_{2}	κ_{2}
κ_{3}	κ_{21}	κ_{12}	κ_{12}	κ_{12}	κ_{3}	κ_{3}	κ_{3}	κ_{21}	κ_{21}	κ_{21}	κ_{12}	κ_{12}	κ_{12}	κ_{3}	κ_{3}	κ_{3}
κ_{4}	κ_{40}	κ_{58}	κ_{76}	κ_{67}	κ_{6}	κ_{24}	κ_{15}	κ_{33}	κ_{51}	κ_{42}	κ_{60}	κ_{78}	κ_{69}	κ_{5}	κ_{23}	κ_{14}
κ_{5}	κ_{50}	κ_{77}	κ_{68}	κ_{59}	κ_{4}	κ_{22}	κ_{13}	κ_{40}	κ_{31}	κ_{49}	κ_{76}	κ_{67}	κ_{58}	κ_{6}	κ_{24}	κ_{15}
κ_{6}	κ_{33}	κ_{69}	κ_{60}	κ_{78}	κ_{5}	κ_{23}	κ_{14}	κ_{50}	κ_{41}	κ_{32}	κ_{63}	κ_{59}	κ_{77}	κ_{4}	κ_{22}	κ_{13}
κ_{7}	κ_{79}	κ_{34}	κ_{43}	κ_{52}	κ_{8}	κ_{17}	κ_{26}	κ_{62}	κ_{71}	κ_{80}	κ_{35}	κ_{44}	κ_{53}	κ_{9}	κ_{18}	κ_{27}
κ_{8}	κ_{62}	κ_{53}	κ_{35}	κ_{44}	κ_{9}	κ_{18}	κ_{27}	κ_{72}	κ_{81}	κ_{63}	κ_{54}	κ_{36}	κ_{45}	κ_{7}	κ_{16}	κ_{25}
κ_{9}	κ_{72}	κ_{45}	κ_{54}	κ_{36}	κ_{7}	κ_{16}	κ_{25}	κ_{79}	κ_{61}	κ_{70}	κ_{43}	κ_{52}	κ_{34}	κ_{8}	κ_{17}	κ_{26}
κ_{10}	κ_{11}	κ_{12}	κ_{12}	κ_{12}	κ_{10}	κ_{10}	κ_{10}	κ_{11}	κ_{11}	κ_{11}	κ_{12}	κ_{12}	κ_{12}	κ_{10}	κ_{10}	κ_{10}
κ_{11}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{21}	κ_{21}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}
κ_{12}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{12}	κ_{12}	κ_{12}
κ_{13}	κ_{50}	κ_{69}	κ_{60}	κ_{78}	κ_{15}	κ_{6}	κ_{24}	κ_{40}	κ_{31}	κ_{49}	κ_{68}	κ_{59}	κ_{77}	κ_{14}	κ_{5}	κ_{23}
κ_{14}	κ_{33}	κ_{58}	κ_{76}	κ_{67}	κ_{13}	κ_{4}	κ_{22}	κ_{4}	κ_{22}	κ_{13}	κ_{60}	κ_{78}	κ_{69}	κ_{15}	κ_{6}	κ_{24}
κ_{15}	κ_{40}	κ_{77}	κ_{68}	κ_{59}	κ_{14}	κ_{5}	κ_{23}	κ_{33}	κ_{51}	κ_{42}	κ_{76}	κ_{67}	κ_{58}	κ_{13}	κ_{4}	κ_{22}
κ_{16}	κ_{62}	κ_{45}	κ_{54}	κ_{36}	κ_{17}	κ_{26}	κ_{8}	κ_{72}	κ_{81}	κ_{63}	κ_{43}	κ_{52}	κ_{34}	κ_{18}	κ_{27}	κ_{9}
κ_{17}	κ_{72}	κ_{34}	κ_{43}	κ_{52}	κ_{18}	κ_{27}	κ_{9}	κ_{79}	κ_{61}	κ_{70}	κ_{35}	κ_{44}	κ_{53}	κ_{16}	κ_{25}	κ_{7}
κ_{18}	κ_{79}	κ_{53}	κ_{35}	κ_{44}	κ_{16}	κ_{25}	κ_{7}	κ_{62}	κ_{71}	κ_{80}	κ_{54}	κ_{36}	κ_{45}	κ_{17}	κ_{26}	κ_{8}
κ_{19}	κ_{21}	κ_{20}	κ_{20}	κ_{20}	κ_{19}	κ_{19}	κ_{19}	κ_{21}	κ_{21}	κ_{21}	κ_{20}	κ_{20}	κ_{20}	κ_{3}	κ_{3}	κ_{3}
κ_{20}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{20}	κ_{20}	κ_{20}
κ_{21}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{11}	κ_{11}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}
κ_{22}	κ_{33}	κ_{77}	κ_{68}	κ_{59}	κ_{24}	κ_{15}	κ_{6}	κ_{50}	κ_{41}	κ_{32}	κ_{76}	κ_{67}	κ_{58}	κ_{23}	κ_{14}	κ_{5}
κ_{23}	κ_{40}	κ_{69}	κ_{60}	κ_{78}	κ_{22}	κ_{13}	κ_{4}	κ_{33}	κ_{51}	κ_{42}	κ_{68}	κ_{59}	κ_{77}	κ_{24}	κ_{15}	κ_{6}
κ_{24}	κ_{50}	κ_{58}	κ_{76}	κ_{67}	κ_{23}	κ_{14}	κ_{5}	κ_{40}	κ_{31}	κ_{49}	κ_{60}	κ_{78}	κ_{69}	κ_{22}	κ_{13}	κ_{4}
κ_{25}	κ_{72}	κ_{53}	κ_{35}	κ_{44}	κ_{26}	κ_{8}	κ_{17}	κ_{81}	κ_{63}	κ_{72}	κ_{53}	κ_{35}	κ_{44}	κ_{27}	κ_{9}	κ_{18}
κ_{26}	κ_{79}	κ_{45}	κ_{54}	κ_{36}	κ_{27}	κ_{9}	κ_{18}	κ_{62}	κ_{71}	κ_{80}	κ_{43}	κ_{52}	κ_{34}	κ_{25}	κ_{7}	κ_{16}
κ_{27}	κ_{62}	κ_{34}	κ_{43}	κ_{52}	κ_{25}	κ_{7}	κ_{16}	κ_{72}	κ_{81}	κ_{63}	κ_{35}	κ_{44}	κ_{53}	κ_{26}	κ_{8}	κ_{17}
κ_{28}	κ_{33}	κ_{34}	κ_{35}	κ_{36}	κ_{37}	κ_{38}	κ_{39}	κ_{40}	κ_{41}	κ_{42}	κ_{43}	κ_{44}	κ_{45}	κ_{46}	κ_{47}	κ_{48}
κ_{29}	κ_{40}	κ_{53}	κ_{54}	κ_{52}	κ_{38}	κ_{39}	κ_{37}	κ_{50}	κ_{51}	κ_{49}	κ_{35}	κ_{36}	κ_{34}	κ_{47}	κ_{48}	κ_{46}
κ_{30}	κ_{50}	κ_{45}	κ_{43}	κ_{44}	κ_{39}	κ_{37}	κ_{38}	κ_{33}	κ_{31}	κ_{32}	κ_{54}	κ_{52}	κ_{53}	κ_{48}	κ_{46}	κ_{47}
κ_{31}	κ_{72}	κ_{1}	κ_{20}	κ_{12}	κ_{42}	κ_{31}	κ_{50}	κ_{72}	κ_{61}	κ_{80}	κ_{12}	κ_{1}	κ_{20}	κ_{50}	κ_{42}	κ_{31}
κ_{32}	κ_{79}	κ_{20}	κ_{12}	κ_{1}	κ_{40}	κ_{32}	κ_{51}	κ_{79}	κ_{71}	κ_{63}	κ_{1}	κ_{20}	κ_{12}	κ_{51}	κ_{40}	κ_{32}
κ_{33}	κ_{62}	κ_{12}	κ_{1}	κ_{20}	κ_{41}	κ_{33}	κ_{49}	κ_{62}	κ_{81}	κ_{70}	κ_{20}	κ_{12}	κ_{1}	κ_{49}	κ_{41}	κ_{33}
κ_{34}	κ_{21}	κ_{58}	κ_{68}	κ_{78}	κ_{44}	κ_{54}	κ_{34}	κ_{11}	κ_{21}	κ_{1}	κ_{68}	κ_{78}	κ_{58}	κ_{54}	κ_{34}	κ_{44}
κ_{35}	κ_{1}	κ_{77}	κ_{60}	κ_{67}	κ_{45}	κ_{52}	κ_{35}	κ_{21}	κ_{1}	κ_{11}	κ_{60}	κ_{67}	κ_{77}	κ_{52}	κ_{35}	κ_{45}
κ_{36}	κ_{11}	κ_{69}	κ_{76}	κ_{59}	κ_{43}	κ_{53}	κ_{36}	κ_{1}	κ_{11}	κ_{21}	κ_{76}	κ_{59}	κ_{69}	κ_{53}	κ_{36}	κ_{43}
κ_{37}	κ_{40}	κ_{45}	κ_{43}	κ_{44}	κ_{46}	κ_{47}	κ_{48}	κ_{50}	κ_{51}	κ_{49}	κ_{54}	κ_{52}	κ_{53}	κ_{28}	κ_{29}	κ_{30}
κ_{38}	κ_{50}	κ_{34}	κ_{35}	κ_{36}	κ_{47}	κ_{48}	κ_{46}	κ_{33}	κ_{31}	κ_{32}	κ_{43}	κ_{44}	κ_{45}	κ_{29}	κ_{30}	κ_{28}
κ_{39}	κ_{33}	κ_{53}	κ_{54}	κ_{52}	κ_{48}	κ_{46}	κ_{47}	κ_{40}	κ_{41}	κ_{42}	κ_{35}	κ_{36}	κ_{34}	κ_{30}	κ_{28}	κ_{29}
κ_{40}	κ_{79}	κ_{12}	κ_{1}	κ_{20}	κ_{51}	κ_{40}	κ_{32}	κ_{79}	κ_{71}	κ_{63}	κ_{20}	κ_{12}	κ_{1}	κ_{32}	κ_{51}	κ_{40}
κ_{41}	κ_{62}	κ_{1}	κ_{20}	κ_{12}	κ_{49}	κ_{41}	κ_{33}	κ_{62}	κ_{81}	κ_{70}	κ_{12}	κ_{1}	κ_{20}	κ_{33}	κ_{49}	κ_{41}
κ_{42}	κ_{72}	κ_{1}	κ_{20}	κ_{12}	κ_{50}	κ_{42}	κ_{31}	κ_{72}	κ_{61}	κ_{80}	κ_{1}	κ_{20}	κ_{12}	κ_{31}	κ_{50}	κ_{42}
κ_{43}	κ_{1}	κ_{69}	κ_{76}	κ_{59}	κ_{53}	κ_{36}	κ_{43}	κ_{21}	κ_{1}	κ_{11}	κ_{76}	κ_{59}	κ_{69}	κ_{36}	κ_{43}	κ_{53}
κ_{44}	κ_{11}	κ_{58}	κ_{68}	κ_{78}	κ_{54}	κ_{34}	κ_{44}	κ_{1}	κ_{11}	κ_{21}	κ_{68}	κ_{78}	κ_{58}	κ_{34}	κ_{44}	κ_{54}
κ_{45}	κ_{21}	κ_{77}	κ_{60}	κ_{67}	κ_{52}	κ_{35}	κ_{45}	κ_{11}	κ_{21}	κ_{1}	κ_{60}	κ_{67}	κ_{77}	κ_{35}	κ_{45}	κ_{52}
κ_{46}	κ_{50}	κ_{53}	κ_{54}	κ_{52}	κ_{28}	κ_{29}	κ_{30}	κ_{33}	κ_{31}	κ_{32}	κ_{33}	κ_{31}	κ_{32}	κ_{37}	κ_{38}	κ_{39}
κ_{47}	κ_{33}	κ_{45}	κ_{43}	κ_{44}	κ_{29}	κ_{30}	κ_{28}	κ_{40}	κ_{41}	κ_{42}	κ_{54}	κ_{52}	κ_{53}	κ_{38}	κ_{39}	κ_{37}
κ_{48}	κ_{40}	κ_{34}	κ_{35}	κ_{36}	κ_{30}	κ_{28}	κ_{29}	κ_{50}	κ_{51}	κ_{49}	κ_{43}	κ_{44}	κ_{45}	κ_{39}	κ_{37}	κ_{38}
κ_{49}	κ_{62}	κ_{20}	κ_{12}	κ_{1}	κ_{33}	κ_{49}	κ_{41}	κ_{62}	κ_{81}	κ_{70}	κ_{1}	κ_{20}	κ_{12}	κ_{41}	κ_{33}	κ_{49}
κ_{50}	κ_{72}	κ_{12}	κ_{1}	κ_{20}	κ_{31}	κ_{50}	κ_{42}	κ_{72}	κ_{61}	κ_{80}	κ_{20}	κ_{12}	κ_{1}	κ_{42}	κ_{31}	κ_{50}
κ_{51}	κ_{79}	κ_{1}	κ_{20}	κ_{12}	κ_{32}	κ_{51}	κ_{40}	κ_{79}	κ_{71}	κ_{63}	κ_{12}	κ_{1}	κ_{20}	κ_{40}	κ_{32}	κ_{51}
κ_{52}	κ_{11}	κ_{77}	κ_{60}	κ_{67}	κ_{35}	κ_{45}	κ_{52}	κ_{1}	κ_{11}	κ_{21}	κ_{60}	κ_{67}	κ_{77}	κ_{45}	κ_{52}	κ_{35}
κ_{53}	κ_{21}	κ_{69}	κ_{76}	κ_{59}	κ_{36}	κ_{43}	κ_{53}	κ_{11}	κ_{21}	κ_{1}	κ_{76}	κ_{59}	κ_{69}	κ_{43}	κ_{53}	κ_{36}
κ_{54}	κ_{1}	κ_{58}	κ_{68}	κ_{78}	κ_{34}	κ_{44}	κ_{54}	κ_{21}	κ_{1}	κ_{11}	κ_{68}	κ_{78}	κ_{58}	κ_{44}	κ_{54}	κ_{34}
κ_{55}	κ_{62}	κ_{58}	κ_{60}	κ_{59}	κ_{73}	κ_{75}	κ_{74}	κ_{79}	κ_{81}	κ_{80}	κ_{76}	κ_{78}	κ_{77}	κ_{64}	κ_{66}	κ_{65}
κ_{56}	κ_{72}	κ_{77}	κ_{76}	κ_{78}	κ_{74}	κ_{73}	κ_{75}	κ_{62}	κ_{61}	κ_{63}	κ_{68}	κ_{67}	κ_{69}	κ_{65}	κ_{64}	κ_{66}
κ_{57}	κ_{79}	κ_{69}	κ_{68}	κ_{67}	κ_{75}	κ_{74}	κ_{73}	κ_{72}	κ_{71}	κ_{70}	κ_{60}	κ_{59}	κ_{58}	κ_{66}	κ_{65}	κ_{64}
κ_{58}	κ_{11}	κ_{34}	κ_{54}	κ_{44}	κ_{78}	κ_{68}	κ_{58}	κ_{21}	κ_{11}	κ_{1}	κ_{54}	κ_{44}	κ_{34}	κ_{68}	κ_{58}	κ_{78}
κ_{59}	κ_{21}	κ_{53}	κ_{43}	κ_{35}	κ_{76}	κ_{69}	κ_{59}	κ_{1}	κ_{21}	κ_{11}	κ_{43}	κ_{36}	κ_{53}	κ_{69}	κ_{59}	κ_{76}
κ_{60}	κ_{1}	κ_{45}	κ_{35}	κ_{52}	κ_{77}	κ_{67}	κ_{60}	κ_{11}	κ_{1}	κ_{21}	κ_{35}	κ_{52}	κ_{45}	κ_{67}	κ_{60}	κ_{77}
κ_{61}	κ_{50}	κ_{1}	κ_{12}	κ_{20}	κ_{80}	κ_{61}	κ_{72}	κ_{50}	κ_{31}	κ_{42}	κ_{20}	κ_{1}	κ_{12}	κ_{72}	κ_{61}	κ_{72}
κ_{62}	κ_{33}	κ_{20}	κ_{1}	κ_{12}	κ_{81}	κ_{62}	κ_{70}	κ_{33}	κ_{41}	κ_{49}	κ_{12}	κ_{20}	κ_{1}	κ_{70}	κ_{81}	κ_{62}
κ_{63}	κ_{40}	κ_{12}	κ_{20}	κ_{1}	κ_{79}	κ_{63}	κ_{71}	κ_{40}	κ_{51}	κ_{32}	κ_{1}	κ_{12}	κ_{20}	κ_{71}	κ_{79}	κ_{63}
κ_{64}	κ_{72}	κ_{69}	κ_{68}	κ_{67}	κ_{55}	κ_{57}	κ_{56}	κ_{62}	κ_{61}	κ_{63}	κ_{60}	κ_{59}	κ_{58}	κ_{73}	κ_{75}	κ_{74}
κ_{65}	κ_{79}	κ_{58}	κ_{60}	κ_{59}	κ_{56}	κ_{55}	κ_{57}	κ_{72}	κ_{71}	κ_{70}	κ_{76}	κ_{78}	κ_{77}	κ_{74}	κ_{73}	κ_{75}
κ_{66}	κ_{62}	κ_{77}	κ_{76}	κ_{78}	κ_{57}	κ_{56}	κ_{55}	κ_{79}	κ_{81}	κ_{80}	κ_{68}	κ_{67}	κ_{69}	κ_{75}	κ_{74}	κ_{73}
κ_{67}	κ_{21}	κ_{45}	κ_{35}	κ_{52}	κ_{60}	κ_{77}	κ_{67}	κ_{1}	κ_{21}	κ_{11}	κ_{35}	κ_{52}	κ_{45}	κ_{77}	κ_{65}	κ_{60}
κ_{68}	κ_{1}	κ_{34}	κ_{54}	κ_{44}	κ_{58}	κ_{78}	κ_{68}	κ_{11}	κ_{1}	κ_{21}	κ_{54}	κ_{44}	κ_{34}	κ_{78}	κ_{68}	κ_{58}
κ_{69}	κ_{11}	κ_{53}	κ_{43}	κ_{36}	κ_{59}	κ_{76}	κ_{69}	κ_{21}	κ_{11}	κ_{1}	κ_{43}	κ_{36}	κ_{53}	κ_{76}	κ_{69}	κ_{59}
κ_{70}	κ_{33}	κ_{12}	κ_{20}	κ_{1}	κ_{62}	κ_{70}	κ_{81}	κ_{33}	κ_{41}	κ_{49}	κ_{1}	κ_{12}	κ_{20}	κ_{81}	κ_{62}	κ_{70}
κ_{71}	κ_{40}	κ_{1}	κ_{12}	κ_{20}	κ_{63}	κ_{71}	κ_{79}	κ_{40}	κ_{51}	κ_{34}	κ_{20}	κ_{1}	κ_{12}	κ_{79}	κ_{63}	κ_{71}
κ_{72}	κ_{50}	κ_{20}	κ_{1}	κ_{12}	κ_{61}	κ_{72}	κ_{80}	κ_{50}	κ_{31}	κ_{42}	κ_{12}	κ_{20}	κ_{1}	κ_{80}	κ_{61}	κ_{72}
κ_{73}	κ_{79}	κ_{77}	κ_{76}	κ_{78}	κ_{64}	κ_{66}	κ_{65}	κ_{72}	κ_{71}	κ_{70}	κ_{68}	κ_{67}	κ_{69}	κ_{55}	κ_{57}	κ_{56}
κ_{74}	κ_{62}	κ_{69}	κ_{68}	κ_{67}	κ_{65}	κ_{64}	κ_{66}	κ_{79}	κ_{81}	κ_{80}	κ_{60}	κ_{59}	κ_{58}	κ_{56}	κ_{55}	κ_{57}
κ_{75}	κ_{72}	κ_{58}	κ_{60}	κ_{59}	κ_{66}	κ_{65}	κ_{64}	κ_{62}	κ_{61}	κ_{63}	κ_{76}	κ_{78}	κ_{77}	κ_{57}	κ_{56}	κ_{55}
κ_{76}	κ_{1}	κ_{53}	κ_{43}	κ_{36}	κ_{69}	κ_{59}	κ_{76}	κ_{11}	κ_{1}	κ_{21}	κ_{43}	κ_{36}	κ_{53}	κ_{59}	κ_{76}	κ_{69}
κ_{77}	κ_{11}	κ_{45}	κ_{35}	κ_{52}	κ_{67}	κ_{60}	κ_{77}	κ_{21}	κ_{11}	κ_{1}	κ_{35}	κ_{52}	κ_{45}	κ_{60}	κ_{77}	κ_{67}
κ_{78}	κ_{21}	κ_{34}	κ_{54}	κ_{44}	κ_{68}	κ_{58}	κ_{78}	κ_{1}	κ_{21}	κ_{11}	κ_{54}	κ_{44}	κ_{34}	κ_{58}	κ_{78}	κ_{68}
κ_{79}	κ_{40}	κ_{20}	κ_{1}	κ_{12}	κ_{71}	κ_{79}	κ_{63}	κ_{40}	κ_{51}	κ_{32}	κ_{12}	κ_{20}	κ_{1}	κ_{63}	κ_{71}	κ_{79}
κ_{80}	κ_{50}	κ_{12}	κ_{20}	κ_{1}	κ_{72}	κ_{80}	κ_{61}	κ_{50}	κ_{31}	κ_{42}	κ_{1}	κ_{12}	κ_{20}	κ_{61}	κ_{72}	κ_{80}
κ_{81}	κ_{33}	κ_{1}	κ_{12}	κ_{20}	κ_{70}	κ_{81}	κ_{62}	κ_{33}	κ_{41}	κ_{49}	κ_{20}	κ_{1}	κ_{12}	κ_{62}	κ_{70}	κ_{81}

*	κ_{49}	κ_{50}	κ_{51}	κ_{52}	κ_{53}	κ_{54}	κ_{55}	κ_{56}	κ_{57}	κ_{58}	κ_{59}	κ_{60}	κ_{61}	κ_{62}	κ_{63}	κ_{64}
κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}
κ_{2}	κ_{11}	κ_{11}	κ_{11}	κ_{20}	κ_{20}	κ_{20}	κ_{3}	κ_{3}	κ_{3}	κ_{12}	κ_{12}	κ_{12}	κ_{21}	κ_{21}	κ_{21}	κ_{3}
κ_{3}	κ_{21}	κ_{21}	κ_{21}	κ_{12}	κ_{12}	κ_{12}	κ_{2}	κ_{2}	κ_{2}	κ_{20}	κ_{20}	κ_{20}	κ_{11}	κ_{11}	κ_{11}	κ_{2}
κ_{4}	κ_{32}	κ_{50}	κ_{41}	κ_{59}	κ_{77}	κ_{68}	κ_{7}	κ_{25}	κ_{16}	κ_{34}	κ_{52}	κ_{43}	κ_{61}	κ_{79}	κ_{70}	κ_{9}
κ_{5}	κ_{42}	κ_{33}	κ_{51}	κ_{78}	κ_{69}	κ_{60}	κ_{9}	κ_{27}	κ_{18}	κ_{45}	κ_{36}	κ_{54}	κ_{81}	κ_{72}	κ_{63}	κ_{8}
κ_{6}	κ_{49}	κ_{40}	κ_{31}	κ_{67}	κ_{58}	κ_{76}	κ_{8}	κ_{26}	κ_{17}	κ_{53}	κ_{44}	κ_{35}	κ_{71}	κ_{62}	κ_{80}	κ_{7}
κ_{7}	κ_{63}	κ_{72}	κ_{81}	κ_{36}	κ_{45}	κ_{54}	κ_{4}	κ_{13}	κ_{22}	κ_{58}	κ_{67}	κ_{76}	κ_{31}	κ_{40}	κ_{49}	κ_{5}
κ_{8}	κ_{70}	κ_{79}	κ_{61}	κ_{52}	κ_{34}	κ_{43}	κ_{6}	κ_{15}	κ_{24}	κ_{69}	κ_{78}	κ_{60}	κ_{51}	κ_{33}	κ_{42}	κ_{4}
κ_{9}	κ_{80}	κ_{62}	κ_{71}	κ_{44}	κ_{53}	κ_{35}	κ_{5}	κ_{14}	κ_{23}	κ_{77}	κ_{59}	κ_{68}	κ_{41}	κ_{50}	κ_{32}	κ_{6}
κ_{10}	κ_{11}	κ_{11}	κ_{11}	κ_{12}	κ_{12}	κ_{12}	κ_{19}	κ_{19}	κ_{19}	κ_{20}	κ_{20}	κ_{20}	κ_{21}	κ_{21}	κ_{21}	κ_{19}
κ_{11}	κ_{21}	κ_{21}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{21}
κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{20}	κ_{20}	κ_{20}	κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}
κ_{13}	κ_{42}	κ_{33}	κ_{51}	κ_{67}	κ_{58}	κ_{76}	κ_{25}	κ_{16}	κ_{7}	κ_{53}	κ_{44}	κ_{35}	κ_{81}	κ_{72}	κ_{63}	κ_{27}
κ_{14}	κ_{6}	κ_{24}	κ_{15}	κ_{59}	κ_{77}	κ_{68}	κ_{27}	κ_{18}	κ_{9}	κ_{34}	κ_{52}	κ_{43}	κ_{71}	κ_{62}	κ_{80}	κ_{26}
κ_{15}	κ_{32}	κ_{50}	κ_{41}	κ_{78}	κ_{69}	κ_{60}	κ_{26}	κ_{17}	κ_{8}	κ_{45}	κ_{36}	κ_{54}	κ_{61}	κ_{79}	κ_{70}	κ_{25}
κ_{16}	κ_{70}	κ_{79}	κ_{61}	κ_{44}	κ_{53}	κ_{35}	κ_{22}	κ_{4}	κ_{13}	κ_{77}	κ_{59}	κ_{68}	κ_{51}	κ_{33}	κ_{42}	κ_{23}
κ_{17}	κ_{71}	κ_{80}	κ_{62}	κ_{36}	κ_{45}	κ_{54}	κ_{24}	κ_{6}	κ_{15}	κ_{58}	κ_{67}	κ_{76}	κ_{41}	κ_{50}	κ_{32}	κ_{22}
κ_{18}	κ_{63}	κ_{72}	κ_{81}	κ_{52}	κ_{34}	κ_{40}	κ_{23}	κ_{5}	κ_{14}	κ_{69}	κ_{78}	κ_{60}	κ_{31}	κ_{45}	κ_{54}	κ_{24}
κ_{19}	κ_{21}	κ_{21}	κ_{21}	κ_{20}	κ_{20}	κ_{20}	κ_{10}	κ_{10}	κ_{10}	κ_{12}	κ_{12}	κ_{12}	κ_{11}	κ_{11}	κ_{11}	κ_{10}
κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{12}	κ_{12}	κ_{12}	κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}
κ_{21}	κ_{11}	κ_{11}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{11}
κ_{22}	κ_{49}	κ_{40}	κ_{31}	κ_{78}	κ_{69}	κ_{60}	κ_{16}	κ_{7}	κ_{25}	κ_{45}	κ_{36}	κ_{54}	κ_{71}	κ_{62}	κ_{80}	κ_{18}
κ_{23}	κ_{31}	κ_{49}	κ_{40}	κ_{67}	κ_{58}	κ_{76}	κ_{18}	κ_{9}	κ_{27}	κ_{53}	κ_{44}	κ_{35}	κ_{61}	κ_{79}	κ_{70}	κ_{17}
κ_{24}	κ_{42}	κ_{33}	κ_{51}	κ_{59}	κ_{77}	κ_{68}	κ_{17}	κ_{8}	κ_{26}	κ_{34}	κ_{52}	κ_{43}	κ_{81}	κ_{72}	κ_{63}	κ_{16}
κ_{25}	κ_{79}	κ_{61}	κ_{70}	κ_{54}	κ_{36}	κ_{45}	κ_{13}	κ_{22}	κ_{4}	κ_{69}	κ_{78}	κ_{60}	κ_{41}	κ_{50}	κ_{32}	κ_{14}
κ_{26}	κ_{63}	κ_{72}	κ_{81}	κ_{44}	κ_{53}	κ_{35}	κ_{15}	κ_{24}	κ_{6}	κ_{77}	κ_{59}	κ_{68}	κ_{31}	κ_{40}	κ_{49}	κ_{13}
κ_{27}	κ_{70}	κ_{79}	κ_{61}	κ_{36}	κ_{45}	κ_{54}	κ_{14}	κ_{23}	κ_{5}	κ_{58}	κ_{67}	κ_{76}	κ_{51}	κ_{33}	κ_{42}	κ_{15}
κ_{28}	κ_{49}	κ_{50}	κ_{51}	κ_{52}	κ_{53}	κ_{54}	κ_{55}	κ_{56}	κ_{57}	κ_{58}	κ_{59}	κ_{60}	κ_{61}	κ_{62}	κ_{63}	κ_{64}
κ_{29}	κ_{32}	κ_{33}	κ_{31}	κ_{44}	κ_{45}	κ_{43}	κ_{57}	κ_{55}	κ_{56}	κ_{69}	κ_{67}	κ_{68}	κ_{81}	κ_{79}	κ_{80}	κ_{66}
κ_{30}	κ_{42}	κ_{40}	κ_{41}	κ_{36}	κ_{34}	κ_{35}	κ_{56}	κ_{57}	κ_{55}	κ_{77}	κ_{78}	κ_{76}	κ_{71}	κ_{72}	κ_{70}	κ_{65}
κ_{31}	κ_{35}	κ_{54}	κ_{43}	κ_{20}	κ_{12}	κ_{1}	κ_{61}	κ_{80}	κ_{72}	κ_{1}	κ_{20}	κ_{12}	κ_{31}	κ_{50}	κ_{42}	κ_{72}
κ_{32}	κ_{63}	κ_{79}	κ_{71}	κ_{57}	κ_{73}	κ_{65}	κ_{63}	κ_{79}	κ_{71}	κ_{12}	κ_{1}	κ_{20}	κ_{51}	κ_{40}	κ_{32}	κ_{71}
κ_{33}	κ_{70}	κ_{62}	κ_{81}	κ_{1}	κ_{20}	κ_{12}	κ_{63}	κ_{81}	κ_{70}	κ_{20}	κ_{12}	κ_{1}	κ_{41}	κ_{33}	κ_{49}	κ_{68}
κ_{34}	κ_{21}	κ_{1}	κ_{11}	κ_{78}	κ_{58}	κ_{68}	κ_{58}	κ_{68}	κ_{78}	κ_{34}	κ_{44}	κ_{54}	κ_{1}	κ_{11}	κ_{21}	κ_{68}
κ_{35}	κ_{1}	κ_{11}	κ_{21}	κ_{67}	κ_{77}	κ_{60}	κ_{60}	κ_{67}	κ_{77}	κ_{45}	κ_{52}	κ_{35}	κ_{21}	κ_{1}	κ_{11}	κ_{67}
κ_{36}	κ_{11}	κ_{21}	κ_{1}	κ_{59}	κ_{69}	κ_{76}	κ_{59}	κ_{69}	κ_{76}	κ_{53}	κ_{36}	κ_{43}	κ_{11}	κ_{21}	κ_{1}	κ_{69}
κ_{37}	κ_{32}	κ_{33}	κ_{31}	κ_{36}	κ_{34}	κ_{35}	κ_{73}	κ_{74}	κ_{75}	κ_{77}	κ_{78}	κ_{76}	κ_{81}	κ_{79}	κ_{80}	κ_{55}
κ_{38}	κ_{42}	κ_{40}	κ_{41}	κ_{52}	κ_{53}	κ_{54}	κ_{75}	κ_{73}	κ_{74}	κ_{58}	κ_{59}	κ_{60}	κ_{71}	κ_{72}	κ_{70}	κ_{57}
κ_{39}	κ_{49}	κ_{50}	κ_{51}	κ_{44}	κ_{45}	κ_{43}	κ_{74}	κ_{75}	κ_{73}	κ_{69}	κ_{67}	κ_{68}	κ_{61}	κ_{62}	κ_{63}	κ_{56}
κ_{40}	κ_{63}	κ_{79}	κ_{71}	κ_{1}	κ_{20}	κ_{12}	κ_{79}	κ_{71}	κ_{63}	κ_{20}	κ_{12}	κ_{1}	κ_{51}	κ_{40}	κ_{32}	κ_{63}
κ_{41}	κ_{70}	κ_{62}	κ_{81}	κ_{20}	κ_{12}	κ_{1}	κ_{81}	κ_{70}	κ_{62}	κ_{1}	κ_{20}	κ_{12}	κ_{41}	κ_{33}	κ_{49}	κ_{62}
κ_{42}	κ_{80}	κ_{72}	κ_{61}	κ_{12}	κ_{1}	κ_{20}	κ_{80}	κ_{72}	κ_{61}	κ_{12}	κ_{1}	κ_{20}	κ_{31}	κ_{50}	κ_{42}	κ_{61}
κ_{43}	κ_{1}	κ_{11}	κ_{21}	κ_{59}	κ_{69}	κ_{76}	κ_{76}	κ_{59}	κ_{69}	κ_{53}	κ_{36}	κ_{43}	κ_{21}	κ_{1}	κ_{11}	κ_{59}
κ_{44}	κ_{11}	κ_{21}	κ_{1}	κ_{78}	κ_{58}	κ_{68}	κ_{78}	κ_{58}	κ_{68}	κ_{34}	κ_{44}	κ_{54}	κ_{11}	κ_{21}	κ_{1}	κ_{58}
κ_{45}	κ_{21}	κ_{1}	κ_{11}	κ_{67}	κ_{77}	κ_{60}	κ_{77}	κ_{60}	κ_{67}	κ_{45}	κ_{52}	κ_{35}	κ_{1}	κ_{11}	κ_{21}	κ_{60}
κ_{46}	κ_{42}	κ_{40}	κ_{41}	κ_{44}	κ_{45}	κ_{43}	κ_{64}	κ_{65}	κ_{66}	κ_{69}	κ_{67}	κ_{68}	κ_{71}	κ_{72}	κ_{70}	κ_{73}
κ_{47}	κ_{49}	κ_{50}	κ_{51}	κ_{36}	κ_{34}	κ_{35}	κ_{66}	κ_{64}	κ_{65}	κ_{77}	κ_{78}	κ_{46}	κ_{61}	κ_{62}	κ_{63}	κ_{75}
κ_{48}	κ_{32}	κ_{33}	κ_{31}	κ_{52}	κ_{53}	κ_{54}	κ_{65}	κ_{66}	κ_{64}	κ_{58}	κ_{59}	κ_{60}	κ_{81}	κ_{79}	κ_{80}	κ_{74}
κ_{49}	κ_{70}	κ_{62}	κ_{81}	κ_{12}	κ_{1}	κ_{20}	κ_{70}	κ_{62}	κ_{81}	κ_{12}	κ_{1}	κ_{20}	κ_{41}	κ_{33}	κ_{49}	κ_{81}
κ_{50}	κ_{80}	κ_{72}	κ_{61}	κ_{1}	κ_{20}	κ_{12}	κ_{72}	κ_{61}	κ_{80}	κ_{20}	κ_{12}	κ_{1}	κ_{31}	κ_{50}	κ_{42}	κ_{80}
κ_{51}	κ_{63}	κ_{79}	κ_{71}	κ_{20}	κ_{12}	κ_{1}	κ_{71}	κ_{63}	κ_{79}	κ_{1}	κ_{20}	κ_{12}	κ_{51}	κ_{40}	κ_{32}	κ_{79}
κ_{52}	κ_{11}	κ_{21}	κ_{1}	κ_{67}	κ_{77}	κ_{60}	κ_{67}	κ_{77}	κ_{60}	κ_{45}	κ_{52}	κ_{35}	κ_{11}	κ_{21}	κ_{1}	κ_{77}
κ_{53}	κ_{21}	κ_{1}	κ_{11}	κ_{59}	κ_{69}	κ_{76}	κ_{69}	κ_{76}	κ_{59}	κ_{53}	κ_{36}	κ_{43}	κ_{1}	κ_{11}	κ_{21}	κ_{76}
κ_{54}	κ_{1}	κ_{11}	κ_{21}	κ_{78}	κ_{58}	κ_{68}	κ_{68}	κ_{78}	κ_{58}	κ_{34}	κ_{44}	κ_{54}	κ_{21}	κ_{1}	κ_{11}	κ_{78}
κ_{55}	κ_{70}	κ_{72}	κ_{71}	κ_{67}	κ_{69}	κ_{68}	κ_{28}	κ_{30}	κ_{29}	κ_{34}	κ_{36}	κ_{35}	κ_{31}	κ_{33}	κ_{32}	κ_{46}
κ_{56}	κ_{80}	κ_{79}	κ_{81}	κ_{59}	κ_{58}	κ_{60}	κ_{30}	κ_{29}	κ_{28}	κ_{45}	κ_{44}	κ_{43}	κ_{51}	κ_{50}	κ_{49}	κ_{48}
κ_{57}	κ_{63}	κ_{62}	κ_{61}	κ_{78}	κ_{77}	κ_{76}	κ_{29}	κ_{28}	κ_{30}	κ_{53}	κ_{52}	κ_{54}	κ_{41}	κ_{40}	κ_{42}	κ_{47}
κ_{58}	κ_{11}	κ_{1}	κ_{21}	κ_{44}	κ_{34}	κ_{54}	κ_{34}	κ_{54}	κ_{44}	κ_{58}	κ_{78}	κ_{68}	κ_{1}	κ_{21}	κ_{11}	κ_{54}
κ_{59}	κ_{21}	κ_{11}	κ_{1}	κ_{36}	κ_{53}	κ_{43}	κ_{36}	κ_{53}	κ_{43}	κ_{69}	κ_{59}	κ_{76}	κ_{21}	κ_{11}	κ_{1}	κ_{53}
κ_{60}	κ_{1}	κ_{21}	κ_{11}	κ_{52}	κ_{45}	κ_{35}	κ_{35}	κ_{52}	κ_{45}	κ_{77}	κ_{67}	κ_{60}	κ_{11}	κ_{1}	κ_{21}	κ_{52}
κ_{61}	κ_{42}	κ_{50}	κ_{31}	κ_{12}	κ_{20}	κ_{1}	κ_{31}	κ_{42}	κ_{50}	κ_{1}	κ_{12}	κ_{20}	κ_{61}	κ_{72}	κ_{80}	κ_{50}
κ_{62}	κ_{49}	κ_{33}	κ_{41}	κ_{1}	κ_{12}	κ_{20}	κ_{33}	κ_{41}	κ_{49}	κ_{12}	κ_{20}	κ_{1}	κ_{81}	κ_{62}	κ_{70}	κ_{49}
κ_{63}	κ_{32}	κ_{40}	κ_{51}	κ_{20}	κ_{1}	κ_{12}	κ_{32}	κ_{40}	κ_{51}	κ_{20}	κ_{1}	κ_{12}	κ_{71}	κ_{79}	κ_{63}	κ_{51}
κ_{64}	κ_{80}	κ_{79}	κ_{81}	κ_{78}	κ_{77}	κ_{76}	κ_{46}	κ_{48}	κ_{47}	κ_{53}	κ_{52}	κ_{54}	κ_{51}	κ_{50}	κ_{49}	κ_{37}
κ_{65}	κ_{63}	κ_{62}	κ_{61}	κ_{67}	κ_{69}	κ_{68}	κ_{48}	κ_{47}	κ_{46}	κ_{34}	κ_{36}	κ_{35}	κ_{41}	κ_{40}	κ_{42}	κ_{39}
κ_{66}	κ_{70}	κ_{72}	κ_{71}	κ_{59}	κ_{58}	κ_{60}	κ_{47}	κ_{46}	κ_{48}	κ_{45}	κ_{44}	κ_{43}	κ_{31}	κ_{33}	κ_{32}	κ_{38}
κ_{67}	κ_{21}	κ_{11}	κ_{1}	κ_{52}	κ_{45}	κ_{35}	κ_{52}	κ_{45}	κ_{35}	κ_{77}	κ_{67}	κ_{60}	κ_{21}	κ_{11}	κ_{1}	κ_{45}
κ_{68}	κ_{1}	κ_{21}	κ_{11}	κ_{44}	κ_{34}	κ_{54}	κ_{54}	κ_{44}	κ_{34}	κ_{58}	κ_{78}	κ_{68}	κ_{11}	κ_{1}	κ_{21}	κ_{44}
κ_{69}	κ_{11}	κ_{1}	κ_{21}	κ_{36}	κ_{53}	κ_{43}	κ_{53}	κ_{43}	κ_{36}	κ_{69}	κ_{59}	κ_{76}	κ_{1}	κ_{21}	κ_{11}	κ_{43}
κ_{70}	κ_{49}	κ_{33}	κ_{41}	κ_{20}	κ_{1}	κ_{12}	κ_{49}	κ_{33}	κ_{41}	κ_{20}	κ_{1}	κ_{12}	κ_{81}	κ_{62}	κ_{70}	κ_{41}
κ_{71}	κ_{32}	κ_{40}	κ_{51}	κ_{12}	κ_{20}	κ_{1}	κ_{51}	κ_{32}	κ_{40}	κ_{1}	κ_{12}	κ_{20}	κ_{71}	κ_{79}	κ_{63}	κ_{40}
κ_{72}	κ_{42}	κ_{50}	κ_{31}	κ_{1}	κ_{12}	κ_{20}	κ_{50}	κ_{31}	κ_{42}	κ_{12}	κ_{20}	κ_{1}	κ_{61}	κ_{72}	κ_{80}	κ_{42}
κ_{73}	κ_{63}	κ_{62}	κ_{61}	κ_{59}	κ_{58}	κ_{60}	κ_{37}	κ_{39}	κ_{38}	κ_{45}	κ_{44}	κ_{43}	κ_{41}	κ_{40}	κ_{42}	κ_{28}
κ_{74}	κ_{70}	κ_{72}	κ_{71}	κ_{78}	κ_{77}	κ_{76}	κ_{39}	κ_{38}	κ_{37}	κ_{53}	κ_{52}	κ_{54}	κ_{31}	κ_{33}	κ_{32}	κ_{30}
κ_{75}	κ_{80}	κ_{79}	κ_{81}	κ_{67}	κ_{69}	κ_{68}	κ_{38}	κ_{37}	κ_{39}	κ_{34}	κ_{36}	κ_{35}	κ_{51}	κ_{50}	κ_{49}	κ_{29}
κ_{76}	κ_{1}	κ_{21}	κ_{11}	κ_{36}	κ_{53}	κ_{43}	κ_{43}	κ_{36}	κ_{53}	κ_{69}	κ_{59}	κ_{76}	κ_{11}	κ_{1}	κ_{21}	κ_{36}
κ_{77}	κ_{11}	κ_{1}	κ_{21}	κ_{52}	κ_{45}	κ_{35}	κ_{45}	κ_{35}	κ_{52}	κ_{77}	κ_{67}	κ_{60}	κ_{1}	κ_{21}	κ_{11}	κ_{35}
κ_{78}	κ_{21}	κ_{11}	κ_{1}	κ_{44}	κ_{34}	κ_{54}	κ_{44}	κ_{34}	κ_{54}	κ_{58}	κ_{78}	κ_{68}	κ_{21}	κ_{11}	κ_{1}	κ_{34}
κ_{79}	κ_{32}	κ_{40}	κ_{51}	κ_{1}	κ_{12}	κ_{20}	κ_{40}	κ_{51}	κ_{32}	κ_{12}	κ_{20}	κ_{1}	κ_{71}	κ_{79}	κ_{63}	κ_{32}
κ_{80}	κ_{42}	κ_{50}	κ_{31}	κ_{20}	κ_{1}	κ_{12}	κ_{42}	κ_{50}	κ_{31}	κ_{20}	κ_{1}	κ_{12}	κ_{61}	κ_{72}	κ_{80}	κ_{31}
κ_{81}	κ_{49}	κ_{33}	κ_{41}	κ_{12}	κ_{20}	κ_{1}	κ_{41}	κ_{49}	κ_{33}	κ_{1}	κ_{12}	κ_{20}	κ_{81}	κ_{62}	κ_{70}	κ_{33}

$*:\kappa_{65}$	κ_{66}	κ_{67}	κ_{68}	κ_{69}	κ_{70}	κ_{71}	κ_{72}	κ_{73}	κ_{74}	κ_{75}	κ_{76}	κ_{77}	κ_{78}	κ_{79}	κ_{80}	κ_{81}
$\kappa_{1}:\kappa_{1}$	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}	κ_{1}
$\kappa_{2}:\kappa_{3}$	κ_{3}	κ_{12}	κ_{12}	κ_{12}	κ_{21}	κ_{21}	κ_{21}	κ_{3}	κ_{3}	κ_{3}	κ_{12}	κ_{12}	κ_{12}	κ_{21}	κ_{21}	κ_{21}
$\kappa_{3}:\kappa_{2}$	κ_{2}	κ_{20}	κ_{20}	κ_{20}	κ_{11}	κ_{11}	κ_{11}	κ_{2}	κ_{2}	κ_{2}	κ_{20}	κ_{20}	κ_{20}	κ_{11}	κ_{11}	κ_{11}
$\kappa_{4}:\kappa_{27}$	κ_{18}	κ_{36}	κ_{54}	κ_{45}	κ_{63}	κ_{81}	κ_{72}	κ_{8}	κ_{26}	κ_{17}	κ_{35}	κ_{53}	κ_{44}	κ_{62}	κ_{80}	κ_{71}
$\kappa_{5}:\kappa_{26}$	κ_{17}	κ_{44}	κ_{35}	κ_{53}	κ_{80}	κ_{71}	κ_{62}	κ_{7}	κ_{25}	κ_{16}	κ_{43}	κ_{34}	κ_{52}	κ_{79}	κ_{70}	κ_{61}
$\kappa_{6}:\kappa_{25}$	κ_{16}	κ_{52}	κ_{43}	κ_{34}	κ_{70}	κ_{61}	κ_{79}	κ_{9}	κ_{27}	κ_{18}	κ_{54}	κ_{45}	κ_{36}	κ_{72}	κ_{63}	κ_{81}
$\kappa_{7}:\kappa_{14}$	κ_{23}	κ_{59}	κ_{68}	κ_{77}	κ_{32}	κ_{41}	κ_{50}	κ_{6}	κ_{15}	κ_{24}	κ_{60}	κ_{69}	κ_{78}	κ_{33}	κ_{42}	κ_{51}
$\kappa_{8}:\kappa_{13}$	κ_{22}	κ_{67}	κ_{76}	κ_{58}	κ_{49}	κ_{31}	κ_{40}	κ_{5}	κ_{14}	κ_{23}	κ_{68}	κ_{77}	κ_{59}	κ_{50}	κ_{32}	κ_{41}
$\kappa_{9}:\kappa_{15}$	κ_{24}	κ_{78}	κ_{60}	κ_{69}	κ_{42}	κ_{51}	κ_{33}	κ_{4}	κ_{13}	κ_{22}	κ_{76}	κ_{58}	κ_{67}	κ_{40}	κ_{49}	κ_{31}
$\kappa_{10}:\kappa_{19}$	κ_{19}	κ_{20}	κ_{20}	κ_{20}	κ_{21}	κ_{21}	κ_{21}	κ_{19}	κ_{19}	κ_{19}	κ_{20}	κ_{20}	κ_{20}	κ_{21}	κ_{21}	κ_{21}
$\kappa_{11}:\kappa_{21}$	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}	κ_{21}	κ_{21}	κ_{21}	κ_{1}	κ_{1}	κ_{1}	κ_{11}	κ_{11}	κ_{11}
$\kappa_{12}:\kappa_{20}$	κ_{20}	κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}	κ_{1}	κ_{20}	κ_{20}	κ_{20}	κ_{12}	κ_{12}	κ_{12}	κ_{1}	κ_{1}	κ_{1}
$\kappa_{13}:\kappa_{18}$	κ_{9}	κ_{43}	κ_{34}	κ_{52}	κ_{71}	κ_{62}	κ_{80}	κ_{26}	κ_{17}	κ_{8}	κ_{54}	κ_{45}	κ_{36}	κ_{79}	κ_{70}	κ_{61}
$\kappa_{14}:\kappa_{17}$	κ_{8}	κ_{36}	κ_{54}	κ_{45}	κ_{70}	κ_{61}	κ_{79}	κ_{25}	κ_{16}	κ_{7}	κ_{53}	κ_{44}	κ_{35}	κ_{70}	κ_{61}	κ_{79}
$\kappa_{15}:\kappa_{16}$	κ_{7}	κ_{44}	κ_{35}	κ_{53}	κ_{63}	κ_{81}	κ_{72}	κ_{27}	κ_{18}	κ_{9}	κ_{43}	κ_{34}	κ_{52}	κ_{62}	κ_{80}	κ_{71}
$\kappa_{16}:\kappa_{5}$	κ_{14}	κ_{78}	κ_{60}	κ_{69}	κ_{49}	κ_{31}	κ_{40}	κ_{24}	κ_{6}	κ_{15}	κ_{76}	κ_{58}	κ_{67}	κ_{50}	κ_{32}	κ_{41}
$\kappa_{17}:\kappa_{4}$	κ_{13}	κ_{59}	κ_{68}	κ_{77}	κ_{42}	κ_{51}	κ_{33}	κ_{23}	κ_{5}	κ_{14}	κ_{60}	κ_{69}	κ_{78}	κ_{40}	κ_{49}	κ_{31}
$\kappa_{18}:\kappa_{6}$	κ_{15}	κ_{67}	κ_{76}	κ_{58}	κ_{32}	κ_{41}	κ_{50}	κ_{22}	κ_{4}	κ_{13}	κ_{68}	κ_{77}	κ_{59}	κ_{33}	κ_{42}	κ_{51}
$\kappa_{19}:\kappa_{10}$	κ_{10}	κ_{12}	κ_{12}	κ_{12}	κ_{11}	κ_{11}	κ_{11}	κ_{10}	κ_{10}	κ_{10}	κ_{12}	κ_{12}	κ_{12}	κ_{11}	κ_{11}	κ_{11}
$\kappa_{20}:\kappa_{12}$	κ_{12}	κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}	κ_{1}	κ_{12}	κ_{12}	κ_{12}	κ_{20}	κ_{20}	κ_{20}	κ_{1}	κ_{1}	κ_{1}
$\kappa_{21}:\kappa_{11}$	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}	κ_{11}	κ_{11}	κ_{11}	κ_{1}	κ_{1}	κ_{1}	κ_{21}	κ_{21}	κ_{21}
$\kappa_{22}:\kappa_{9}$	κ_{27}	κ_{44}	κ_{35}	κ_{53}	κ_{70}	κ_{61}	κ_{79}	κ_{17}	κ_{8}	κ_{26}	κ_{43}	κ_{34}	κ_{52}	κ_{72}	κ_{63}	κ_{81}
$\kappa_{23}:\kappa_{8}$	κ_{26}	κ_{52}	κ_{43}	κ_{34}	κ_{63}	κ_{81}	κ_{72}	κ_{16}	κ_{7}	κ_{25}	κ_{54}	κ_{45}	κ_{36}	κ_{62}	κ_{80}	κ_{71}
$\kappa_{24}:\kappa_{7}$	κ_{25}	κ_{36}	κ_{54}	κ_{45}	κ_{80}	κ_{71}	κ_{62}	κ_{18}	κ_{9}	κ_{27}	κ_{35}	κ_{53}	κ_{44}	κ_{79}	κ_{70}	κ_{61}
$\kappa_{25}:\kappa_{23}$	κ_{5}	κ_{68}	κ_{77}	κ_{59}	κ_{40}	κ_{49}	κ_{31}	κ_{15}	κ_{24}	κ_{6}	κ_{67}	κ_{76}	κ_{58}	κ_{42}	κ_{51}	κ_{33}
$\kappa_{26}:\kappa_{22}$	κ_{4}	κ_{78}	κ_{60}	κ_{69}	κ_{32}	κ_{41}	κ_{50}	κ_{14}	κ_{23}	κ_{5}	κ_{76}	κ_{58}	κ_{67}	κ_{33}	κ_{42}	κ_{51}
$\kappa_{27}:\kappa_{24}$	κ_{6}	κ_{59}	κ_{68}	κ_{77}	κ_{49}	κ_{31}	κ_{40}	κ_{13}	κ_{22}	κ_{4}	κ_{60}	κ_{69}	κ_{78}	κ_{50}	κ_{32}	κ_{41}
$\kappa_{28}:\kappa_{65}$	κ_{66}	κ_{67}	κ_{68}	κ_{69}	κ_{70}	κ_{71}	κ_{72}	κ_{73}	κ_{74}	κ_{75}	κ_{76}	κ_{77}	κ_{78}	κ_{79}	κ_{80}	κ_{81}
$\kappa_{29}:\kappa_{64}$	κ_{65}	κ_{78}	κ_{76}	κ_{77}	κ_{63}	κ_{61}	κ_{62}	κ_{75}	κ_{73}	κ_{74}	κ_{60}	κ_{58}	κ_{59}	κ_{72}	κ_{70}	κ_{71}
$\kappa_{30}:\kappa_{66}$	κ_{64}	κ_{59}	κ_{60}	κ_{58}	κ_{80}	κ_{81}	κ_{79}	κ_{74}	κ_{75}	κ_{73}	κ_{68}	κ_{69}	κ_{67}	κ_{62}	κ_{63}	κ_{61}
$\kappa_{31}:\kappa_{61}$	κ_{80}	κ_{12}	κ_{1}	κ_{20}	κ_{42}	κ_{31}	κ_{50}	κ_{80}	κ_{72}	κ_{61}	κ_{20}	κ_{12}	κ_{1}	κ_{50}	κ_{42}	κ_{31}
$\kappa_{32}:\kappa_{63}$	κ_{79}	κ_{20}	κ_{12}	κ_{1}	κ_{32}	κ_{51}	κ_{40}	κ_{79}	κ_{71}	κ_{63}	κ_{1}	κ_{20}	κ_{12}	κ_{40}	κ_{32}	κ_{51}
$\kappa_{33}:\kappa_{60}$	κ_{76}	κ_{8}	κ_{27}	κ_{16}	κ_{47}	κ_{39}	κ_{28}	κ_{81}	κ_{70}	κ_{62}	κ_{12}	κ_{1}	κ_{20}	κ_{33}	κ_{49}	κ_{41}
$\kappa_{34}:\kappa_{78}$	κ_{58}	κ_{44}	κ_{54}	κ_{34}	κ_{11}	κ_{21}	κ_{1}	κ_{78}	κ_{58}	κ_{68}	κ_{54}	κ_{34}	κ_{44}	κ_{21}	κ_{1}	κ_{11}
$\kappa_{35}:\kappa_{77}$	κ_{60}	κ_{52}	κ_{35}	κ_{45}	κ_{1}	κ_{11}	κ_{21}	κ_{77}	κ_{60}	κ_{67}	κ_{35}	κ_{45}	κ_{52}	κ_{11}	κ_{21}	κ_{1}
$\kappa_{36}:\kappa_{76}$	κ_{59}	κ_{36}	κ_{43}	κ_{53}	κ_{21}	κ_{1}	κ_{11}	κ_{76}	κ_{59}	κ_{69}	κ_{43}	κ_{53}	κ_{36}	κ_{1}	κ_{11}	κ_{21}
$\kappa_{37}:\kappa_{56}$	κ_{57}	κ_{59}	κ_{60}	κ_{58}	κ_{63}	κ_{61}	κ_{62}	κ_{64}	κ_{65}	κ_{66}	κ_{68}	κ_{69}	κ_{67}	κ_{72}	κ_{70}	κ_{71}
$\kappa_{38}:\kappa_{55}$	κ_{56}	κ_{67}	κ_{68}	κ_{69}	κ_{80}	κ_{81}	κ_{79}	κ_{66}	κ_{64}	κ_{65}	κ_{76}	κ_{77}	κ_{78}	κ_{62}	κ_{63}	κ_{61}
$\kappa_{39}:\kappa_{57}$	κ_{55}	κ_{78}	κ_{76}	κ_{77}	κ_{70}	κ_{71}	κ_{72}	κ_{65}	κ_{66}	κ_{64}	κ_{60}	κ_{58}	κ_{59}	κ_{79}	κ_{80}	κ_{81}
$\kappa_{40}:\kappa_{79}$	κ_{71}	κ_{1}	κ_{20}	κ_{12}	κ_{32}	κ_{51}	κ_{40}	κ_{71}	κ_{63}	κ_{79}	κ_{12}	κ_{1}	κ_{20}	κ_{40}	κ_{32}	κ_{51}
$\kappa_{41}:\kappa_{81}$	κ_{70}	κ_{12}	κ_{1}	κ_{20}	κ_{49}	κ_{41}	κ_{33}	κ_{70}	κ_{62}	κ_{81}	κ_{20}	κ_{12}	κ_{1}	κ_{33}	κ_{49}	κ_{41}
$\kappa_{42}:\kappa_{80}$	κ_{72}	κ_{20}	κ_{12}	κ_{1}	κ_{42}	κ_{31}	κ_{50}	κ_{72}	κ_{61}	κ_{80}	κ_{1}	κ_{20}	κ_{12}	κ_{50}	κ_{42}	κ_{31}
$\kappa_{43}:\kappa_{69}$	κ_{76}	κ_{36}	κ_{43}	κ_{53}	κ_{1}	κ_{11}	κ_{21}	κ_{69}	κ_{76}	κ_{59}	κ_{43}	κ_{53}	κ_{36}	κ_{11}	κ_{21}	κ_{1}
$\kappa_{44}:\kappa_{68}$	κ_{78}	κ_{44}	κ_{54}	κ_{34}	κ_{21}	κ_{1}	κ_{11}	κ_{68}	κ_{78}	κ_{58}	κ_{54}	κ_{34}	κ_{44}	κ_{1}	κ_{11}	κ_{21}
$\kappa_{45}:\kappa_{67}$	κ_{77}	κ_{52}	κ_{35}	κ_{45}	κ_{11}	κ_{21}	κ_{1}	κ_{67}	κ_{77}	κ_{60}	κ_{35}	κ_{45}	κ_{52}	κ_{21}	κ_{1}	κ_{11}
$\kappa_{46}:\kappa_{74}$	κ_{75}	κ_{78}	κ_{76}	κ_{77}	κ_{80}	κ_{81}	κ_{79}	κ_{55}	κ_{56}	κ_{57}	κ_{60}	κ_{58}	κ_{59}	κ_{62}	κ_{63}	κ_{61}
$\kappa_{47}:\kappa_{73}$	κ_{74}	κ_{59}	κ_{60}	κ_{58}	κ_{70}	κ_{71}	κ_{72}	κ_{57}	κ_{55}	κ_{56}	κ_{68}	κ_{69}	κ_{67}	κ_{79}	κ_{80}	κ_{81}
$\kappa_{48}:\kappa_{75}$	κ_{73}	κ_{67}	κ_{68}	κ_{69}	κ_{63}	κ_{61}	κ_{62}	κ_{56}	κ_{57}	κ_{55}	κ_{76}	κ_{77}	κ_{78}	κ_{72}	κ_{70}	κ_{71}
$\kappa_{49}:\kappa_{70}$	κ_{62}	κ_{20}	κ_{12}	κ_{1}	κ_{49}	κ_{41}	κ_{33}	κ_{62}	κ_{81}	κ_{70}	κ_{1}	κ_{20}	κ_{12}	κ_{33}	κ_{49}	κ_{41}
$\kappa_{50}:\kappa_{72}$	κ_{61}	κ_{1}	κ_{20}	κ_{12}	κ_{42}	κ_{31}	κ_{50}	κ_{61}	κ_{80}	κ_{72}	κ_{12}	κ_{1}	κ_{20}	κ_{50}	κ_{42}	κ_{31}
$\kappa_{51}:\kappa_{71}$	κ_{63}	κ_{12}	κ_{1}	κ_{20}	κ_{32}	κ_{51}	κ_{40}	κ_{63}	κ_{79}	κ_{71}	κ_{20}	κ_{12}	κ_{1}	κ_{40}	κ_{32}	κ_{51}
$\kappa_{52}:\kappa_{60}$	κ_{67}	κ_{52}	κ_{35}	κ_{45}	κ_{21}	κ_{1}	κ_{11}	κ_{60}	κ_{67}	κ_{77}	κ_{35}	κ_{45}	κ_{52}	κ_{1}	κ_{11}	κ_{21}
$\kappa_{53}:\kappa_{59}$	κ_{69}	κ_{36}	κ_{43}	κ_{53}	κ_{11}	κ_{21}	κ_{1}	κ_{59}	κ_{69}	κ_{76}	κ_{43}	κ_{53}	κ_{36}	κ_{21}	κ_{1}	κ_{11}
$\kappa_{54}:\kappa_{58}$	κ_{68}	κ_{44}	κ_{54}	κ_{34}	κ_{1}	κ_{11}	κ_{21}	κ_{58}	κ_{68}	κ_{78}	κ_{54}	κ_{34}	κ_{44}	κ_{11}	κ_{21}	κ_{1}
$\kappa_{55}:\kappa_{48}$	κ_{47}	κ_{52}	κ_{54}	κ_{53}	κ_{49}	κ_{51}	κ_{50}	κ_{37}	κ_{39}	κ_{38}	κ_{43}	κ_{45}	κ_{44}	κ_{40}	κ_{42}	κ_{41}
$\kappa_{56}:\kappa_{47}$	κ_{46}	κ_{36}	κ_{35}	κ_{34}	κ_{42}	κ_{41}	κ_{40}	κ_{39}	κ_{38}	κ_{37}	κ_{54}	κ_{53}	κ_{52}	κ_{33}	κ_{32}	κ_{31}
$\kappa_{57}:\kappa_{46}$	κ_{48}	κ_{44}	κ_{43}	κ_{45}	κ_{32}	κ_{31}	κ_{33}	κ_{38}	κ_{37}	κ_{39}	κ_{35}	κ_{34}	κ_{36}	κ_{50}	κ_{49}	κ_{51}
$\kappa_{58}:\kappa_{44}$	κ_{34}	κ_{78}	κ_{68}	κ_{58}	κ_{21}	κ_{11}	κ_{1}	κ_{44}	κ_{34}	κ_{54}	κ_{68}	κ_{58}	κ_{78}	κ_{11}	κ_{1}	κ_{21}
$\kappa_{59}:\kappa_{43}$	κ_{36}	κ_{59}	κ_{76}	κ_{69}	κ_{11}	κ_{1}	κ_{21}	κ_{43}	κ_{36}	κ_{53}	κ_{76}	κ_{69}	κ_{59}	κ_{1}	κ_{21}	κ_{11}
$\kappa_{60}:\kappa_{45}$	κ_{35}	κ_{67}	κ_{60}	κ_{77}	κ_{1}	κ_{21}	κ_{11}	κ_{45}	κ_{35}	κ_{52}	κ_{60}	κ_{77}	κ_{67}	κ_{21}	κ_{11}	κ_{1}
$\kappa_{61}:\kappa_{31}$	κ_{42}	κ_{20}	κ_{1}	κ_{12}	κ_{80}	κ_{61}	κ_{72}	κ_{42}	κ_{50}	κ_{31}	κ_{12}	κ_{20}	κ_{1}	κ_{72}	κ_{80}	κ_{61}
$\kappa_{62}:\kappa_{33}$	κ_{41}	κ_{1}	κ_{12}	κ_{20}	κ_{70}	κ_{81}	κ_{62}	κ_{41}	κ_{49}	κ_{33}	κ_{20}	κ_{1}	κ_{12}	κ_{62}	κ_{70}	κ_{81}
$\kappa_{63}:\kappa_{32}$	κ_{40}	κ_{12}	κ_{20}	κ_{1}	κ_{63}	κ_{71}	κ_{79}	κ_{40}	κ_{51}	κ_{32}	κ_{1}	κ_{12}	κ_{20}	κ_{79}	κ_{63}	κ_{71}
$\kappa_{64}:\kappa_{39}$	κ_{38}	κ_{44}	κ_{43}	κ_{45}	κ_{42}	κ_{41}	κ_{40}	κ_{28}	κ_{30}	κ_{29}	κ_{35}	κ_{34}	κ_{36}	κ_{33}	κ_{32}	κ_{31}
$\kappa_{65}:\kappa_{38}$	κ_{37}	κ_{52}	κ_{54}	κ_{53}	κ_{32}	κ_{31}	κ_{33}	κ_{30}	κ_{29}	κ_{28}	κ_{43}	κ_{45}	κ_{44}	κ_{50}	κ_{49}	κ_{51}
$\kappa_{66}:\kappa_{37}$	κ_{39}	κ_{36}	κ_{35}	κ_{34}	κ_{49}	κ_{51}	κ_{50}	κ_{29}	κ_{28}	κ_{30}	κ_{54}	κ_{53}	κ_{52}	κ_{40}	κ_{42}	κ_{41}
$\kappa_{67}:\kappa_{35}$	κ_{52}	κ_{67}	κ_{60}	κ_{77}	κ_{11}	κ_{1}	κ_{21}	κ_{35}	κ_{52}	κ_{45}	κ_{60}	κ_{77}	κ_{67}	κ_{1}	κ_{21}	κ_{11}
$\kappa_{68}:\kappa_{34}$	κ_{54}	κ_{78}	κ_{68}	κ_{58}	κ_{1}	κ_{21}	κ_{11}	κ_{34}	κ_{54}	κ_{44}	κ_{68}	κ_{58}	κ_{78}	κ_{21}	κ_{11}	κ_{1}
$\kappa_{69}:\kappa_{36}$	κ_{53}	κ_{59}	κ_{76}	κ_{69}	κ_{21}	κ_{11}	κ_{1}	κ_{36}	κ_{53}	κ_{43}	κ_{76}	κ_{69}	κ_{59}	κ_{11}	κ_{1}	κ_{21}
$\kappa_{70}:\kappa_{49}$	κ_{33}	κ_{12}	κ_{20}	κ_{1}	κ_{70}	κ_{81}	κ_{62}	κ_{33}	κ_{41}	κ_{49}	κ_{1}	κ_{12}	κ_{20}	κ_{62}	κ_{70}	κ_{81}
$\kappa_{71}:\kappa_{51}$	κ_{32}	κ_{20}	κ_{1}	κ_{12}	κ_{63}	κ_{71}	κ_{79}	κ_{32}	κ_{40}	κ_{51}	κ_{12}	κ_{20}	κ_{1}	κ_{79}	κ_{63}	κ_{71}
$\kappa_{72}:\kappa_{50}$	κ_{31}	κ_{1}	κ_{12}	κ_{20}	κ_{80}	κ_{61}	κ_{72}	κ_{31}	κ_{42}	κ_{50}	κ_{20}	κ_{1}	κ_{12}	κ_{72}	κ_{80}	κ_{61}
$\kappa_{73}:\kappa_{30}$	κ_{29}	κ_{36}	κ_{35}	κ_{34}	κ_{32}	κ_{31}	κ_{33}	κ_{46}	κ_{48}	κ_{47}	κ_{54}	κ_{53}	κ_{52}	κ_{50}	κ_{49}	κ_{51}
$\kappa_{74}:\kappa_{29}$	κ_{28}	κ_{44}	κ_{43}	κ_{45}	κ_{49}	κ_{51}	κ_{50}	κ_{48}	κ_{47}	κ_{46}	κ_{35}	κ_{34}	κ_{36}	κ_{40}	κ_{42}	κ_{41}
$\kappa_{75}:\kappa_{28}$	κ_{30}	κ_{52}	κ_{54}	κ_{53}	κ_{42}	κ_{41}	κ_{40}	κ_{47}	κ_{46}	κ_{48}	κ_{43}	κ_{45}	κ_{44}	κ_{33}	κ_{32}	κ_{31}
$\kappa_{76}:\kappa_{53}$	κ_{43}	κ_{59}	κ_{76}	κ_{69}	κ_{1}	κ_{21}	κ_{11}	κ_{53}	κ_{43}	κ_{36}	κ_{76}	κ_{69}	κ_{59}	κ_{21}	κ_{11}	κ_{1}
$\kappa_{77}:\kappa_{52}$	κ_{45}	κ_{67}	κ_{60}	κ_{77}	κ_{21}	κ_{11}	κ_{1}	κ_{52}	κ_{45}	κ_{35}	κ_{60}	κ_{77}	κ_{67}	κ_{11}	κ_{1}	κ_{21}
$\kappa_{78}:\kappa_{54}$	κ_{44}	κ_{78}	κ_{68}	κ_{58}	κ_{11}	κ_{1}	κ_{21}	κ_{54}	κ_{44}	κ_{34}	κ_{68}	κ_{58}	κ_{78}	κ_{1}	κ_{21}	κ_{11}
$\kappa_{79}:\kappa_{40}$	κ_{51}	κ_{1}	κ_{12}	κ_{20}	κ_{63}	κ_{71}	κ_{79}	κ_{51}	κ_{32}	κ_{40}	κ_{20}	κ_{1}	κ_{12}	κ_{79}	κ_{63}	κ_{71}
$\kappa_{80}:\kappa_{42}$	κ_{50}	κ_{12}	κ_{20}	κ_{1}	κ_{80}	κ_{61}	κ_{72}	κ_{50}	κ_{31}	κ_{42}	κ_{1}	κ_{12}	κ_{20}	κ_{72}	κ_{80}	κ_{61}
$\kappa_{81}:\kappa_{41}$	κ_{49}	κ_{20}	κ_{1}	κ_{12}	κ_{70}	κ_{81}	κ_{62}	κ_{49}	κ_{33}	κ_{41}	κ_{12}	κ_{20}	κ_{1}	κ_{62}	κ_{70}	κ_{81}

El grupo multiplicativo de las unidades de I_3, denotado por I_3^* es de orden 36. El grupo I_3^* está representado mediante la siguiente tabla, la cual se compone de dos tablas más pequeñas:

*	κ_{4}	κ_{5}	κ_{6}	κ_{7}	κ_{8}	κ_{9}	κ_{13}	κ_{14}	κ_{15}	κ_{16}	κ_{17}	κ_{18}	κ_{22}	κ_{23}	κ_{24}	κ_{25}	κ_{26}	κ_{27}
κ_{4}	κ_{28}	κ_{46}	κ_{37}	κ_{55}	κ_{73}	κ_{64}	κ_{30}	κ_{48}	κ_{39}	κ_{57}	κ_{75}	κ_{66}	κ_{29}	κ_{47}	κ_{38}	κ_{56}	κ_{74}	κ_{65}
κ_{5}	κ_{37}	κ_{28}	κ_{46}	κ_{73}	κ_{64}	κ_{55}	κ_{39}	κ_{30}	κ_{48}	κ_{75}	κ_{66}	κ_{57}	κ_{38}	κ_{29}	κ_{47}	κ_{74}	κ_{65}	κ_{56}
κ_{6}	κ_{46}	κ_{37}	κ_{28}	κ_{64}	κ_{55}	κ_{73}	κ_{48}	κ_{39}	κ_{30}	κ_{66}	κ_{57}	κ_{75}	κ_{47}	κ_{38}	κ_{29}	κ_{65}	κ_{56}	κ_{74}
κ_{7}	κ_{55}	κ_{64}	κ_{73}	κ_{28}	κ_{37}	κ_{46}	κ_{56}	κ_{65}	κ_{74}	κ_{29}	κ_{38}	κ_{47}	κ_{57}	κ_{66}	κ_{75}	κ_{30}	κ_{39}	κ_{48}
κ_{8}	κ_{64}	κ_{73}	κ_{55}	κ_{46}	κ_{28}	κ_{37}	κ_{65}	κ_{74}	κ_{56}	κ_{47}	κ_{29}	κ_{38}	κ_{66}	κ_{75}	κ_{57}	κ_{48}	κ_{30}	κ_{39}
κ_{9}	κ_{73}	κ_{55}	κ_{64}	κ_{37}	κ_{46}	κ_{28}	κ_{74}	κ_{56}	κ_{65}	κ_{38}	κ_{47}	κ_{29}	κ_{75}	κ_{57}	κ_{66}	κ_{39}	κ_{48}	κ_{30}
κ_{13}	κ_{29}	κ_{47}	κ_{38}	κ_{57}	κ_{75}	κ_{66}	κ_{28}	κ_{46}	κ_{37}	κ_{56}	κ_{74}	κ_{65}	κ_{30}	κ_{48}	κ_{39}	κ_{55}	κ_{73}	κ_{64}
κ_{14}	κ_{38}	κ_{29}	κ_{47}	κ_{75}	κ_{66}	κ_{57}	κ_{37}	κ_{28}	κ_{46}	κ_{74}	κ_{65}	κ_{56}	κ_{39}	κ_{30}	κ_{48}	κ_{73}	κ_{64}	κ_{55}
κ_{15}	κ_{47}	κ_{38}	κ_{29}	κ_{66}	κ_{57}	κ_{75}	κ_{46}	κ_{37}	κ_{28}	κ_{65}	κ_{56}	κ_{74}	κ_{48}	κ_{39}	κ_{30}	κ_{64}	κ_{55}	κ_{73}
κ_{16}	κ_{56}	κ_{65}	κ_{74}	κ_{30}	κ_{39}	κ_{48}	κ_{57}	κ_{66}	κ_{75}	κ_{28}	κ_{37}	κ_{46}	κ_{55}	κ_{64}	κ_{73}	κ_{29}	κ_{38}	κ_{47}
κ_{17}	κ_{65}	κ_{74}	κ_{56}	κ_{48}	κ_{30}	κ_{39}	κ_{66}	κ_{75}	κ_{57}	κ_{46}	κ_{28}	κ_{37}	κ_{64}	κ_{73}	κ_{55}	κ_{47}	κ_{29}	κ_{38}
κ_{18}	κ_{74}	κ_{56}	κ_{65}	κ_{39}	κ_{48}	κ_{30}	κ_{75}	κ_{57}	κ_{66}	κ_{37}	κ_{46}	κ_{28}	κ_{73}	κ_{55}	κ_{64}	κ_{38}	κ_{47}	κ_{29}
κ_{22}	κ_{30}	κ_{48}	κ_{39}	κ_{56}	κ_{74}	κ_{65}	κ_{29}	κ_{47}	κ_{38}	κ_{55}	κ_{73}	κ_{64}	κ_{28}	κ_{46}	κ_{37}	κ_{57}	κ_{75}	κ_{66}
κ_{23}	κ_{39}	κ_{30}	κ_{48}	κ_{74}	κ_{65}	κ_{56}	κ_{38}	κ_{29}	κ_{47}	κ_{73}	κ_{64}	κ_{55}	κ_{37}	κ_{28}	κ_{46}	κ_{75}	κ_{66}	κ_{57}
κ_{24}	κ_{48}	κ_{39}	κ_{30}	κ_{65}	κ_{56}	κ_{74}	κ_{47}	κ_{38}	κ_{29}	κ_{64}	κ_{55}	κ_{73}	κ_{46}	κ_{37}	κ_{28}	κ_{66}	κ_{57}	κ_{75}
κ_{25}	κ_{57}	κ_{66}	κ_{75}	κ_{29}	κ_{38}	κ_{47}	κ_{55}	κ_{64}	κ_{73}	κ_{30}	κ_{39}	κ_{48}	κ_{56}	κ_{65}	κ_{74}	κ_{28}	κ_{37}	κ_{46}
κ_{26}	κ_{66}	κ_{75}	κ_{57}	κ_{47}	κ_{29}	κ_{38}	κ_{64}	κ_{73}	κ_{55}	κ_{48}	κ_{30}	κ_{39}	κ_{65}	κ_{74}	κ_{56}	κ_{46}	κ_{28}	κ_{37}
κ_{27}	κ_{75}	κ_{57}	κ_{66}	κ_{38}	κ_{47}	κ_{29}	κ_{73}	κ_{55}	κ_{64}	κ_{39}	κ_{48}	κ_{30}	κ_{74}	κ_{56}	κ_{65}	κ_{37}	κ_{46}	κ_{28}
κ_{28}	κ_{4}	κ_{5}	κ_{6}	κ_{7}	κ_{8}	κ_{9}	κ_{13}	κ_{14}	κ_{15}	κ_{16}	κ_{17}	κ_{18}	κ_{22}	κ_{23}	κ_{24}	κ_{25}	κ_{26}	κ_{27}
κ_{29}	κ_{13}	κ_{14}	κ_{15}	κ_{25}	κ_{26}	κ_{27}	κ_{22}	κ_{23}	κ_{24}	κ_{7}	κ_{8}	κ_{9}	κ_{4}	κ_{5}	κ_{6}	κ_{16}	κ_{17}	κ_{18}
κ_{30}	κ_{22}	κ_{23}	κ_{24}	κ_{16}	κ_{17}	κ_{18}	κ_{4}	κ_{5}	κ_{6}	κ_{25}	κ_{26}	κ_{27}	κ_{13}	κ_{14}	κ_{15}	κ_{7}	κ_{8}	κ_{9}
κ_{37}	κ_{5}	κ_{6}	κ_{4}	κ_{9}	κ_{7}	κ_{8}	κ_{14}	κ_{15}	κ_{13}	κ_{18}	κ_{16}	κ_{17}	κ_{23}	κ_{24}	κ_{22}	κ_{27}	κ_{25}	κ_{26}
κ_{38}	κ_{14}	κ_{15}	κ_{13}	κ_{27}	κ_{25}	κ_{26}	κ_{23}	κ_{24}	κ_{22}	κ_{9}	κ_{7}	κ_{8}	κ_{5}	κ_{6}	κ_{4}	κ_{18}	κ_{16}	κ_{17}
κ_{39}	κ_{23}	κ_{24}	κ_{22}	κ_{18}	κ_{16}	κ_{17}	κ_{5}	κ_{6}	κ_{4}	κ_{27}	κ_{25}	κ_{26}	κ_{14}	κ_{15}	κ_{13}	κ_{9}	κ_{7}	κ_{8}
κ_{46}	κ_{6}	κ_{4}	κ_{5}	κ_{8}	κ_{9}	κ_{7}	κ_{15}	κ_{13}	κ_{14}	κ_{17}	κ_{18}	κ_{16}	κ_{24}	κ_{22}	κ_{23}	κ_{26}	κ_{27}	κ_{25}
κ_{47}	κ_{15}	κ_{13}	κ_{14}	κ_{26}	κ_{27}	κ_{25}	κ_{24}	κ_{22}	κ_{23}	κ_{8}	κ_{9}	κ_{7}	κ_{6}	κ_{4}	κ_{5}	κ_{17}	κ_{18}	κ_{16}
κ_{48}	κ_{24}	κ_{22}	κ_{23}	κ_{17}	κ_{18}	κ_{16}	κ_{6}	κ_{4}	κ_{5}	κ_{26}	κ_{27}	κ_{25}	κ_{15}	κ_{13}	κ_{14}	κ_{8}	κ_{9}	κ_{7}
κ_{55}	κ_{7}	κ_{9}	κ_{8}	κ_{4}	κ_{6}	κ_{5}	κ_{25}	κ_{27}	κ_{26}	κ_{22}	κ_{24}	κ_{23}	κ_{16}	κ_{18}	κ_{17}	κ_{13}	κ_{15}	κ_{14}
κ_{56}	κ_{16}	κ_{18}	κ_{17}	κ_{22}	κ_{24}	κ_{23}	κ_{7}	κ_{9}	κ_{8}	κ_{13}	κ_{15}	κ_{14}	κ_{25}	κ_{27}	κ_{26}	κ_{4}	κ_{6}	κ_{5}
κ_{57}	κ_{25}	κ_{27}	κ_{26}	κ_{13}	κ_{15}	κ_{14}	κ_{16}	κ_{18}	κ_{17}	κ_{4}	κ_{6}	κ_{5}	κ_{7}	κ_{9}	κ_{8}	κ_{22}	κ_{24}	κ_{23}
κ_{64}	κ_{8}	κ_{7}	κ_{9}	κ_{6}	κ_{5}	κ_{4}	κ_{26}	κ_{25}	κ_{27}	κ_{24}	κ_{23}	κ_{22}	κ_{17}	κ_{16}	κ_{18}	κ_{15}	κ_{14}	κ_{13}
κ_{65}	κ_{17}	κ_{16}	κ_{18}	κ_{24}	κ_{23}	κ_{22}	κ_{8}	κ_{7}	κ_{9}	κ_{15}	κ_{14}	κ_{13}	κ_{26}	κ_{25}	κ_{27}	κ_{6}	κ_{5}	κ_{4}
κ_{66}	κ_{26}	κ_{25}	κ_{27}	κ_{15}	κ_{14}	κ_{13}	κ_{17}	κ_{16}	κ_{18}	κ_{6}	κ_{5}	κ_{4}	κ_{8}	κ_{7}	κ_{9}	κ_{24}	κ_{23}	κ_{22}
κ_{73}	κ_{9}	κ_{8}	κ_{7}	κ_{5}	κ_{4}	κ_{6}	κ_{27}	κ_{26}	κ_{25}	κ_{23}	κ_{22}	κ_{24}	κ_{18}	κ_{17}	κ_{16}	κ_{14}	κ_{13}	κ_{15}
κ_{74}	κ_{18}	κ_{17}	κ_{16}	κ_{23}	κ_{22}	κ_{24}	κ_{9}	κ_{8}	κ_{7}	κ_{14}	κ_{13}	κ_{15}	κ_{27}	κ_{26}	κ_{25}	κ_{5}	κ_{4}	κ_{6}
κ_{75}	κ_{27}	κ_{26}	κ_{25}	κ_{14}	κ_{13}	κ_{15}	κ_{18}	κ_{17}	κ_{16}	κ_{5}	κ_{4}	κ_{6}	κ_{9}	κ_{8}	κ_{7}	κ_{23}	κ_{22}	κ_{24}

*	κ_{28}	κ_{29}	κ_{30}	κ_{37}	κ_{38}	κ_{39}	κ_{46}	κ_{47}	κ_{48}	κ_{55}	κ_{56}	κ_{57}	κ_{64}	κ_{65}	κ_{66}	κ_{73}	κ_{74}	κ_{75}
κ_{4}	κ_{4}	κ_{22}	κ_{13}	κ_{6}	κ_{24}	κ_{15}	κ_{5}	κ_{23}	κ_{14}	κ_{7}	κ_{25}	κ_{16}	κ_{9}	κ_{27}	κ_{18}	κ_{8}	κ_{26}	κ_{17}
κ_{5}	κ_{5}	κ_{23}	κ_{14}	κ_{4}	κ_{22}	κ_{13}	κ_{6}	κ_{24}	κ_{15}	κ_{9}	κ_{27}	κ_{18}	κ_{8}	κ_{26}	κ_{17}	κ_{7}	κ_{25}	κ_{16}
κ_{6}	κ_{6}	κ_{24}	κ_{15}	κ_{5}	κ_{23}	κ_{14}	κ_{4}	κ_{22}	κ_{13}	κ_{8}	κ_{26}	κ_{17}	κ_{7}	κ_{25}	κ_{16}	κ_{9}	κ_{27}	κ_{18}
κ_{7}	κ_{7}	κ_{16}	κ_{25}	κ_{8}	κ_{17}	κ_{26}	κ_{9}	κ_{18}	κ_{27}	κ_{4}	κ_{13}	κ_{22}	κ_{5}	κ_{14}	κ_{23}	κ_{6}	κ_{15}	κ_{24}
κ_{8}	κ_{8}	κ_{17}	κ_{26}	κ_{9}	κ_{18}	κ_{27}	κ_{7}	κ_{16}	κ_{25}	κ_{6}	κ_{15}	κ_{24}	κ_{4}	κ_{13}	κ_{22}	κ_{5}	κ_{14}	κ_{23}
κ_{9}	κ_{9}	κ_{18}	κ_{27}	κ_{7}	κ_{16}	κ_{25}	κ_{8}	κ_{17}	κ_{26}	κ_{5}	κ_{14}	κ_{23}	κ_{6}	κ_{15}	κ_{24}	κ_{4}	κ_{13}	κ_{22}
κ_{13}	κ_{13}	κ_{4}	κ_{22}	κ_{15}	κ_{6}	κ_{24}	κ_{14}	κ_{5}	κ_{23}	κ_{25}	κ_{16}	κ_{7}	κ_{27}	κ_{18}	κ_{9}	κ_{26}	κ_{17}	κ_{8}
κ_{14}	κ_{14}	κ_{5}	κ_{23}	κ_{13}	κ_{4}	κ_{22}	κ_{15}	κ_{6}	κ_{24}	κ_{27}	κ_{18}	κ_{9}	κ_{26}	κ_{17}	κ_{8}	κ_{25}	κ_{16}	κ_{7}
κ_{15}	κ_{15}	κ_{6}	κ_{24}	κ_{14}	κ_{5}	κ_{23}	κ_{13}	κ_{4}	κ_{22}	κ_{26}	κ_{17}	κ_{8}	κ_{25}	κ_{16}	κ_{7}	κ_{27}	κ_{18}	κ_{9}
κ_{16}	κ_{16}	κ_{25}	κ_{7}	κ_{17}	κ_{26}	κ_{8}	κ_{18}	κ_{27}	κ_{9}	κ_{22}	κ_{4}	κ_{13}	κ_{23}	κ_{5}	κ_{14}	κ_{24}	κ_{6}	κ_{15}
κ_{17}	κ_{17}	κ_{26}	κ_{8}	κ_{18}	κ_{27}	κ_{9}	κ_{16}	κ_{25}	κ_{7}	κ_{24}	κ_{6}	κ_{15}	κ_{22}	κ_{4}	κ_{13}	κ_{23}	κ_{5}	κ_{14}
κ_{18}	κ_{18}	κ_{27}	κ_{9}	κ_{16}	κ_{25}	κ_{7}	κ_{17}	κ_{26}	κ_{8}	κ_{23}	κ_{5}	κ_{14}	κ_{24}	κ_{6}	κ_{15}	κ_{22}	κ_{4}	κ_{13}
κ_{22}	κ_{22}	κ_{13}	κ_{4}	κ_{24}	κ_{15}	κ_{6}	κ_{23}	κ_{14}	κ_{5}	κ_{16}	κ_{7}	κ_{25}	κ_{18}	κ_{9}	κ_{27}	κ_{17}	κ_{8}	κ_{26}
κ_{23}	κ_{23}	κ_{14}	κ_{5}	κ_{22}	κ_{13}	κ_{4}	κ_{24}	κ_{15}	κ_{6}	κ_{18}	κ_{9}	κ_{27}	κ_{17}	κ_{8}	κ_{26}	κ_{16}	κ_{7}	κ_{25}
κ_{24}	κ_{24}	κ_{15}	κ_{6}	κ_{23}	κ_{14}	κ_{5}	κ_{22}	κ_{13}	κ_{4}	κ_{17}	κ_{8}	κ_{26}	κ_{16}	κ_{7}	κ_{25}	κ_{18}	κ_{9}	κ_{27}
κ_{25}	κ_{25}	κ_{7}	κ_{16}	κ_{26}	κ_{8}	κ_{17}	κ_{27}	κ_{9}	κ_{18}	κ_{13}	κ_{22}	κ_{4}	κ_{14}	κ_{23}	κ_{5}	κ_{15}	κ_{24}	κ_{6}
κ_{26}	κ_{26}	κ_{8}	κ_{17}	κ_{27}	κ_{9}	κ_{18}	κ_{25}	κ_{7}	κ_{16}	κ_{15}	κ_{24}	κ_{6}	κ_{13}	κ_{22}	κ_{4}	κ_{14}	κ_{23}	κ_{5}
κ_{27}	κ_{27}	κ_{9}	κ_{18}	κ_{25}	κ_{7}	κ_{16}	κ_{26}	κ_{8}	κ_{17}	κ_{14}	κ_{23}	κ_{5}	κ_{15}	κ_{24}	κ_{6}	κ_{13}	κ_{22}	κ_{4}
κ_{28}	κ_{28}	κ_{29}	κ_{30}	κ_{37}	κ_{38}	κ_{39}	κ_{46}	κ_{47}	κ_{48}	κ_{55}	κ_{56}	κ_{57}	κ_{64}	κ_{65}	κ_{66}	κ_{73}	κ_{74}	κ_{75}
κ_{29}	κ_{29}	κ_{30}	κ_{28}	κ_{38}	κ_{39}	κ_{37}	κ_{47}	κ_{48}	κ_{46}	κ_{57}	κ_{55}	κ_{56}	κ_{66}	κ_{64}	κ_{65}	κ_{75}	κ_{73}	κ_{74}
κ_{30}	κ_{30}	κ_{28}	κ_{29}	κ_{39}	κ_{37}	κ_{38}	κ_{48}	κ_{46}	κ_{47}	κ_{56}	κ_{57}	κ_{55}	κ_{65}	κ_{66}	κ_{64}	κ_{74}	κ_{75}	κ_{73}
κ_{37}	κ_{37}	κ_{38}	κ_{39}	κ_{46}	κ_{47}	κ_{48}	κ_{28}	κ_{29}	κ_{30}	κ_{73}	κ_{74}	κ_{75}	κ_{55}	κ_{56}	κ_{57}	κ_{64}	κ_{65}	κ_{66}
κ_{38}	κ_{38}	κ_{39}	κ_{37}	κ_{47}	κ_{48}	κ_{46}	κ_{29}	κ_{30}	κ_{28}	κ_{75}	κ_{73}	κ_{74}	κ_{57}	κ_{55}	κ_{56}	κ_{66}	κ_{64}	κ_{65}
κ_{39}	κ_{39}	κ_{37}	κ_{38}	κ_{48}	κ_{46}	κ_{47}	κ_{30}	κ_{28}	κ_{29}	κ_{74}	κ_{75}	κ_{73}	κ_{56}	κ_{57}	κ_{55}	κ_{65}	κ_{66}	κ_{64}
κ_{46}	κ_{46}	κ_{47}	κ_{48}	κ_{28}	κ_{29}	κ_{30}	κ_{37}	κ_{38}	κ_{39}	κ_{64}	κ_{65}	κ_{66}	κ_{73}	κ_{74}	κ_{75}	κ_{55}	κ_{56}	κ_{57}
κ_{47}	κ_{47}	κ_{48}	κ_{46}	κ_{29}	κ_{30}	κ_{28}	κ_{38}	κ_{39}	κ_{37}	κ_{66}	κ_{64}	κ_{65}	κ_{75}	κ_{73}	κ_{74}	κ_{57}	κ_{55}	κ_{56}
κ_{48}	κ_{48}	κ_{46}	κ_{47}	κ_{30}	κ_{28}	κ_{29}	κ_{39}	κ_{37}	κ_{38}	κ_{65}	κ_{66}	κ_{64}	κ_{74}	κ_{75}	κ_{73}	κ_{56}	κ_{57}	κ_{55}
κ_{55}	κ_{55}	κ_{57}	κ_{56}	κ_{73}	κ_{75}	κ_{74}	κ_{64}	κ_{66}	κ_{65}	κ_{28}	κ_{30}	κ_{29}	κ_{46}	κ_{48}	κ_{47}	κ_{37}	κ_{39}	κ_{38}
κ_{56}	κ_{56}	κ_{55}	κ_{57}	κ_{74}	κ_{73}	κ_{75}	κ_{65}	κ_{64}	κ_{66}	κ_{30}	κ_{29}	κ_{28}	κ_{48}	κ_{47}	κ_{46}	κ_{39}	κ_{38}	κ_{37}
κ_{57}	κ_{57}	κ_{56}	κ_{55}	κ_{75}	κ_{74}	κ_{73}	κ_{66}	κ_{65}	κ_{64}	κ_{29}	κ_{28}	κ_{30}	κ_{47}	κ_{46}	κ_{48}	κ_{38}	κ_{37}	κ_{39}
κ_{64}	κ_{64}	κ_{66}	κ_{65}	κ_{55}	κ_{57}	κ_{56}	κ_{73}	κ_{75}	κ_{74}	κ_{46}	κ_{48}	κ_{47}	κ_{37}	κ_{39}	κ_{38}	κ_{28}	κ_{30}	κ_{29}
κ_{65}	κ_{65}	κ_{64}	κ_{66}	κ_{56}	κ_{55}	κ_{57}	κ_{74}	κ_{73}	κ_{75}	κ_{48}	κ_{47}	κ_{46}	κ_{39}	κ_{38}	κ_{37}	κ_{30}	κ_{29}	κ_{28}
κ_{66}	κ_{66}	κ_{65}	κ_{64}	κ_{57}	κ_{56}	κ_{55}	κ_{75}	κ_{74}	κ_{73}	κ_{47}	κ_{46}	κ_{48}	κ_{38}	κ_{37}	κ_{39}	κ_{29}	κ_{28}	κ_{30}
κ_{73}	κ_{73}	κ_{75}	κ_{74}	κ_{64}	κ_{66}	κ_{65}	κ_{55}	κ_{57}	κ_{56}	κ_{37}	κ_{39}	κ_{38}	κ_{28}	κ_{30}	κ_{29}	κ_{46}	κ_{48}	κ_{47}
κ_{74}	κ_{74}	κ_{73}	κ_{75}	κ_{65}	κ_{64}	κ_{66}	κ_{56}	κ_{55}	κ_{57}	κ_{39}	κ_{38}	κ_{37}	κ_{30}	κ_{29}	κ_{28}	κ_{48}	κ_{47}	κ_{46}
κ_{75}	κ_{75}	κ_{74}	κ_{73}	κ_{66}	κ_{65}	κ_{64}	κ_{57}	κ_{56}	κ_{55}	κ_{38}	κ_{37}	κ_{39}	κ_{29}	κ_{28}	κ_{30}	κ_{47}	κ_{46}	κ_{48}

Debido a que la inclusión de las tablas de multiplicación de los productos cruzados J_3, E_3 y F_3 requeriría 15 páginas, es conveniente mencionar los elementos de los que consiste cada uno y dar explícitamente la multiplicación correspondiente. Sin embargo, vamos a exhibir cada uno de los grupos de unidades $\overline{J}_3^*$, E_3^* y $\overline{F}_3^*$ incluyendo en una página mediante dos tablas.

La multiplicación en J_3 es dada por

$$a\omega_{g^i} b\omega_{g^j} = ab^{g^i} f(g^i, g^j)\omega_{g^{i+j}} = \begin{cases} ab^{g^i}\omega_{g^{i+j}} & \text{si } i+j<2 \\ 2ab^{g^i}\omega_1 & \text{si } i+j=2\,, \end{cases} \text{, donde } 0 \leq i,j < 2.$$

Se puede renombrar los elementos de J_3 preservando el mismo orden por
$J_3 = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5, \upsilon_6, \upsilon_7, \upsilon_8, \upsilon_9, \upsilon_{10}, \upsilon_{11}, \upsilon_{12}, \upsilon_{13}, \upsilon_{14}, \upsilon_{15}, \upsilon_{16}, \upsilon_{17}, \upsilon_{18}, \upsilon_{19}, \upsilon_{20},$
$\upsilon_{21}, \upsilon_{22}, \upsilon_{23}, \upsilon_{24}, \upsilon_{25}, \upsilon_{26}, \upsilon_{27}, \upsilon_{28}, \upsilon_{29}, \upsilon_{30}, \upsilon_{31}, \upsilon_{32}, \upsilon_{33}, \upsilon_{34}, \upsilon_{35}, \upsilon_{36}, \upsilon_{37}, \upsilon_{38}, \upsilon_{39}, \upsilon_{40},$
$\upsilon_{41}, \upsilon_{42}, \upsilon_{43}, \upsilon_{44}, \upsilon_{45}, \upsilon_{46}, \upsilon_{47}, \upsilon_{48}, \upsilon_{49}, \upsilon_{50}, \upsilon_{51}, \upsilon_{52}, \upsilon_{53}, \upsilon_{54}, \upsilon_{55}, \upsilon_{56}, \upsilon_{57}, \upsilon_{58}, \upsilon_{59}, \upsilon_{60},$
$\upsilon_{61}, \upsilon_{62}, \upsilon_{63}, \upsilon_{64}, \upsilon_{65}, \upsilon_{66}, \upsilon_{67}, \upsilon_{68}, \upsilon_{69}, \upsilon_{70}, \upsilon_{71}, \upsilon_{72}, \upsilon_{73}, \upsilon_{74}, \upsilon_{75}, \upsilon_{76}, \upsilon_{77}, \upsilon_{78}, \upsilon_{79}, \upsilon_{80}, \upsilon_{81}\}$

Para construir su tabla de multiplicación como en el caso anterior consistiendo de 5 tablas.

Por otro lado, si se define la acción de C_2 sobre A por $g \cdot 1 = 1$ y $g \cdot x = x$ (acción trivial). Para cada $\alpha \in A^* = \{1, 2, 1+x, 2+x, 1+2x, 2+2x\}$, la aplicación $f_\alpha : C_2 \times C_2 \to A^*$ definida por

$$f_\alpha(g^i, g^j) = \begin{cases} 1 & \text{si } i+j<2 \\ \alpha & \text{si } i+j \geq 2\,, \end{cases}$$

donde $0 \leq i,j < 2$, es un 2-cociclo para el producto cruzado $A \rtimes_{f_\alpha} C_2$.
Además, si $\alpha \in \mathbb{Z}_3^*$ se tiene que $\alpha = 1$, $\alpha = 2$. En particular, para $\alpha = 1$, se denota $A \rtimes_{f_\alpha} C_2$ por E_3; para $\alpha = 2$, se denota $A \rtimes_{f_\alpha} C_2$ por F_3.

La multiplicación en E_3 es dada por $a\omega_{g^i} b\omega_{g^j} = ab\omega_{g^{i+j}}$.
Análogamente, se puede renombrar los elementos de E_3 preservando el mismo orden por
$E_3 = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \rho_9, \rho_{10}, \rho_{11}, \rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{16}, \rho_{17}, \rho_{18}, \rho_{19}, \rho_{20},$
$\rho_{21}, \rho_{22}, \rho_{23}, \rho_{24}, \rho_{25}, \rho_{26}, \rho_{27}, \rho_{28}, \rho_{29}, \rho_{30}, \rho_{31}, \rho_{32}, \rho_{33}, \rho_{34}, \rho_{35}, \rho_{36}, \rho_{37}, \rho_{38}, \rho_{39}, \rho_{40},$
$\rho_{41}, \rho_{42}, \rho_{43}, \rho_{44}, \rho_{45}, \rho_{46}, \rho_{47}, \rho_{48}, \rho_{49}, \rho_{50}, \rho_{51}, \rho_{52}, \rho_{53}, \rho_{54}, \rho_{55}, \rho_{56}, \rho_{57}, \rho_{58}, \rho_{59}, \rho_{60},$
$\rho_{61}, \rho_{62}, \rho_{63}, \rho_{64}, \rho_{65}, \rho_{66}, \rho_{67}, \rho_{68}, \rho_{69}, \rho_{70}, \rho_{71}, \rho_{72}, \rho_{73}, \rho_{74}, \rho_{75}, \rho_{76}, \rho_{77}, \rho_{78}, \rho_{79}, \rho_{80}, \rho_{81}\}$ para construir su tabla de multiplicación consistiendo de 5 tablas.

La multiplicación en F_3 es dada por

$$a\omega_{g^i} b\omega_{g^j} = \begin{cases} ab\omega_{g^{i+j}} & \text{si } i+j<2 \\ 2ab\omega_1 & \text{si } i+j=2\,, \end{cases} \text{, donde } 0 \leq i,j < 2.$$

Se puede renombrar los elementos de F_3 preservando el mismo orden por
$F_3 = \{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8, \eta_9, \eta_{10}, \eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}, \eta_{15}, \eta_{16}, \eta_{17}, \eta_{18}, \eta_{19}, \eta_{20},$
$\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}, \eta_{25}, \eta_{26}, \eta_{27}, \eta_{28}, \eta_{29}, \eta_{30}, \eta_{31}, \eta_{32}, \eta_{33}, \eta_{34}, \eta_{35}, \eta_{36}, \eta_{37}, \eta_{38}, \eta_{39}, \eta_{40},$
$\eta_{41}, \eta_{42}, \eta_{43}, \eta_{44}, \eta_{45}, \eta_{46}, \eta_{47}, \eta_{48}, \eta_{49}, \eta_{50}, \eta_{51}, \eta_{52}, \eta_{53}, \eta_{54}, \eta_{55}, \eta_{56}, \eta_{57}, \eta_{58}, \eta_{59}, \eta_{60},$
$\eta_{61}, \eta_{62}, \eta_{63}, \eta_{64}, \eta_{65}, \eta_{66}, \eta_{67}, \eta_{68}, \eta_{69}, \eta_{70}, \eta_{71}, \eta_{72}, \eta_{73}, \eta_{74}, \eta_{75}, \eta_{76}, \eta_{77}, \eta_{78}, \eta_{79}, \eta_{80}, \eta_{81}\}$ para construir su tabla de multiplicación consistiendo de 5 tablas.

Para explicar los grupos $\overline{J}_3^*$ y $\overline{F}_3^*$ necesitamos un nuevo concepto sobre los elementos de J_3 y F_3.

Definición 3.1. *El discriminante de un elemento $v = (a+bx)\omega_1 + (c+dx)\omega_g$ de J_3 se define como* $\Delta(v) = a^2 - 2c^2$. Usando este concepto, $J_3^* = \{v \in J_3 \ : \ \Delta(v) \neq 0\}$.

Proposición 3.1. *Sea $\overline{J}_3^* = \{v \in J_3^* \ : \ \Delta(v) = 1\}$, entonces $\overline{J}_3^*$ es un subgrupo de J_3^*.*

Demostración: Claramente, $\omega_1 \in J_3^*$ está en $\overline{J}_3^*$. Además, $\overline{J}_3^*$ es cerrado bajo la operación y la inversa. Sean v y $v' \in \overline{J}_3^*$ como en la Definición 3.1, entonces

$$vv' = [(aa' + 2cc') + (ba' + ab' + 2dc' - 2cd')x]\omega_1 + [(ac' + ca') + (bc' + ad' + da' - cb')x]\omega_g.$$

Luego, $\Delta(vv') = [(aa' + 2cc')^2 - 2(ac' + ca')^2 = (a^2 - 2c^2)(a'^2 - 2c'^2) = (1)(1) = 1$.
Por otro lado, dado $v \in \overline{J}_3^*$, resolviendo la ecuación $vv' = \omega_1$ se obtiene que $\Delta(v') = a'^2 - 2c'^2 = 1$. $\square$

El discriminante de un elemento $v = (a+bx)\omega_1 + (c+dx)\omega_g$ de F_3 también se define como $\Delta(v) = a^2 - 2c^2$. Por lo tanto, $\overline{F}_3^* = \{v \in F_3^* \ : \ \Delta(v) = 1\}$ es un grupo de unidades de F_3.

Observación 3.1. *Se cumple la propiedad $[\Delta(v)]^2 = 1$ para todo $v \in J_3^*$. Observando esta propiedad, vemos que el polinomio $X^2 + 1$ es irreducible en $\mathbb{Z}_3[X]$.*

El grupo $\overline{J}_3^*$ está representado mediante la siguiente tabla, la cual se compone de dos tablas más pequeñas:

$*$	v_4	v_5	v_6	v_7	v_8	v_9	v_{13}	v_{14}	v_{15}	v_{16}	v_{17}	v_{18}	v_{22}	v_{23}	v_{24}	v_{25}	v_{26}	v_{27}
v_4	v_{55}	v_{64}	v_{73}	v_{28}	v_{37}	v_{46}	v_{57}	v_{66}	v_{75}	v_{30}	v_{39}	v_{48}	v_{56}	v_{65}	v_{74}	v_{29}	v_{38}	v_{47}
v_5	v_{73}	v_{55}	v_{64}	v_{37}	v_{46}	v_{28}	v_{75}	v_{57}	v_{66}	v_{39}	v_{48}	v_{30}	v_{74}	v_{56}	v_{65}	v_{38}	v_{47}	v_{29}
v_6	v_{64}	v_{73}	v_{55}	v_{46}	v_{28}	v_{37}	v_{66}	v_{75}	v_{57}	v_{48}	v_{30}	v_{39}	v_{65}	v_{74}	v_{56}	v_{47}	v_{29}	v_{38}
v_7	v_{28}	v_{46}	v_{37}	v_{55}	v_{73}	v_{64}	v_{29}	v_{47}	v_{38}	v_{56}	v_{74}	v_{65}	v_{30}	v_{48}	v_{39}	v_{57}	v_{75}	v_{66}
v_8	v_{46}	v_{37}	v_{28}	v_{64}	v_{55}	v_{73}	v_{47}	v_{38}	v_{29}	v_{65}	v_{56}	v_{74}	v_{48}	v_{39}	v_{30}	v_{66}	v_{57}	v_{75}
v_9	v_{37}	v_{28}	v_{46}	v_{73}	v_{64}	v_{55}	v_{38}	v_{29}	v_{47}	v_{74}	v_{65}	v_{56}	v_{39}	v_{30}	v_{48}	v_{75}	v_{66}	v_{57}
v_{13}	v_{56}	v_{65}	v_{74}	v_{30}	v_{39}	v_{48}	v_{55}	v_{64}	v_{73}	v_{29}	v_{38}	v_{47}	v_{57}	v_{66}	v_{75}	v_{28}	v_{37}	v_{46}
v_{14}	v_{74}	v_{56}	v_{65}	v_{39}	v_{48}	v_{30}	v_{73}	v_{55}	v_{64}	v_{38}	v_{47}	v_{29}	v_{75}	v_{57}	v_{66}	v_{37}	v_{46}	v_{28}
v_{15}	v_{65}	v_{74}	v_{56}	v_{48}	v_{30}	v_{39}	v_{64}	v_{73}	v_{55}	v_{47}	v_{29}	v_{38}	v_{66}	v_{75}	v_{57}	v_{46}	v_{28}	v_{37}
v_{16}	v_{29}	v_{47}	v_{38}	v_{57}	v_{75}	v_{66}	v_{30}	v_{48}	v_{39}	v_{55}	v_{73}	v_{64}	v_{28}	v_{46}	v_{37}	v_{56}	v_{74}	v_{65}
v_{17}	v_{47}	v_{38}	v_{29}	v_{66}	v_{57}	v_{75}	v_{48}	v_{39}	v_{30}	v_{64}	v_{55}	v_{73}	v_{46}	v_{37}	v_{28}	v_{65}	v_{56}	v_{74}
v_{18}	v_{38}	v_{29}	v_{47}	v_{75}	v_{66}	v_{57}	v_{39}	v_{30}	v_{48}	v_{73}	v_{64}	v_{55}	v_{37}	v_{28}	v_{46}	v_{74}	v_{65}	v_{56}
v_{22}	v_{57}	v_{66}	v_{75}	v_{29}	v_{38}	v_{47}	v_{56}	v_{65}	v_{74}	v_{28}	v_{37}	v_{46}	v_{55}	v_{64}	v_{73}	v_{30}	v_{39}	v_{48}
v_{23}	v_{75}	v_{57}	v_{66}	v_{39}	v_{47}	v_{29}	v_{74}	v_{56}	v_{65}	v_{37}	v_{46}	v_{28}	v_{73}	v_{55}	v_{64}	v_{39}	v_{48}	v_{30}
v_{24}	v_{66}	v_{75}	v_{57}	v_{47}	v_{29}	v_{38}	v_{65}	v_{74}	v_{56}	v_{46}	v_{28}	v_{37}	v_{64}	v_{73}	v_{55}	v_{48}	v_{30}	v_{39}
v_{25}	v_{30}	v_{48}	v_{39}	v_{56}	v_{74}	v_{65}	v_{28}	v_{46}	v_{37}	v_{57}	v_{75}	v_{66}	v_{29}	v_{47}	v_{38}	v_{55}	v_{73}	v_{64}
v_{26}	v_{48}	v_{39}	v_{30}	v_{65}	v_{56}	v_{74}	v_{46}	v_{37}	v_{28}	v_{66}	v_{57}	v_{75}	v_{47}	v_{38}	v_{29}	v_{64}	v_{55}	v_{73}
v_{27}	v_{39}	v_{30}	v_{48}	v_{74}	v_{65}	v_{56}	v_{37}	v_{28}	v_{46}	v_{75}	v_{66}	v_{57}	v_{38}	v_{29}	v_{47}	v_{73}	v_{64}	v_{55}
v_{28}	v_4	v_5	v_6	v_7	v_8	v_9	v_{13}	v_{14}	v_{15}	v_{16}	v_{17}	v_{18}	v_{22}	v_{23}	v_{24}	v_{25}	v_{26}	v_{27}
v_{29}	v_{22}	v_{23}	v_{24}	v_{16}	v_{17}	v_{18}	v_4	v_5	v_6	v_{25}	v_{26}	v_{27}	v_{13}	v_{14}	v_{15}	v_7	v_8	v_9
v_{30}	v_{13}	v_{14}	v_{15}	v_{25}	v_{26}	v_{27}	v_{22}	v_{23}	v_{24}	v_7	v_8	v_9	v_4	v_5	v_6	v_{16}	v_{17}	v_{18}
v_{37}	v_5	v_6	v_4	v_9	v_7	v_8	v_{14}	v_{15}	v_{13}	v_{18}	v_{16}	v_{17}	v_{23}	v_{24}	v_{22}	v_{27}	v_{25}	v_{26}
v_{38}	v_{23}	v_{24}	v_{22}	v_{18}	v_{16}	v_{17}	v_5	v_6	v_4	v_{27}	v_{25}	v_{26}	v_{14}	v_{15}	v_{13}	v_9	v_7	v_8
v_{39}	v_{14}	v_{15}	v_{13}	v_{27}	v_{25}	v_{26}	v_{23}	v_{24}	v_{22}	v_9	v_7	v_8	v_5	v_6	v_4	v_{18}	v_{16}	v_{17}
v_{46}	v_6	v_4	v_5	v_8	v_9	v_7	v_{15}	v_{13}	v_{14}	v_{17}	v_{18}	v_{16}	v_{24}	v_{22}	v_{23}	v_{26}	v_{27}	v_{25}
v_{47}	v_{24}	v_{22}	v_{23}	v_{17}	v_{18}	v_{16}	v_6	v_4	v_5	v_{26}	v_{27}	v_{25}	v_{15}	v_{13}	v_{14}	v_8	v_9	v_7
v_{48}	v_{15}	v_{13}	v_{14}	v_{26}	v_{27}	v_{25}	v_{24}	v_{22}	v_{23}	v_8	v_9	v_7	v_6	v_4	v_5	v_{17}	v_{18}	v_{16}
v_{55}	v_7	v_9	v_8	v_4	v_6	v_5	v_{25}	v_{27}	v_{26}	v_{22}	v_{24}	v_{23}	v_{16}	v_{18}	v_{17}	v_{13}	v_{15}	v_{14}
v_{56}	v_{25}	v_{27}	v_{26}	v_{13}	v_{15}	v_{14}	v_{16}	v_{18}	v_{17}	v_4	v_6	v_5	v_7	v_9	v_8	v_{22}	v_{24}	v_{23}
v_{57}	v_{16}	v_{18}	v_{17}	v_{22}	v_{24}	v_{23}	v_7	v_9	v_8	v_{13}	v_{15}	v_{14}	v_{25}	v_{27}	v_{26}	v_4	v_6	v_5
v_{64}	v_8	v_7	v_9	v_6	v_5	v_4	v_{26}	v_{25}	v_{27}	v_{24}	v_{23}	v_{22}	v_{17}	v_{16}	v_{18}	v_{15}	v_{14}	v_{13}
v_{65}	v_{26}	v_{25}	v_{27}	v_{15}	v_{14}	v_{13}	v_{17}	v_{16}	v_{18}	v_6	v_5	v_4	v_8	v_7	v_9	v_{24}	v_{23}	v_{22}
v_{66}	v_{17}	v_{16}	v_{18}	v_{24}	v_{23}	v_{22}	v_8	v_7	v_9	v_{15}	v_{14}	v_{13}	v_{26}	v_{25}	v_{27}	v_6	v_5	v_4
v_{73}	v_9	v_8	v_7	v_5	v_4	v_6	v_{27}	v_{26}	v_{25}	v_{23}	v_{22}	v_{24}	v_{18}	v_{17}	v_{16}	v_{14}	v_{13}	v_{15}
v_{74}	v_{27}	v_{26}	v_{25}	v_{14}	v_{13}	v_{15}	v_{18}	v_{17}	v_{16}	v_5	v_4	v_6	v_9	v_8	v_7	v_{23}	v_{22}	v_{24}
v_{75}	v_{18}	v_{17}	v_{16}	v_{23}	v_{22}	v_{24}	v_9	v_8	v_7	v_{14}	v_{13}	v_{15}	v_{27}	v_{26}	v_{25}	v_5	v_4	v_6

$*:v_{28}$	v_{29}	v_{30}	v_{37}	v_{38}	v_{39}	v_{46}	v_{47}	v_{48}	v_{55}	v_{56}	v_{57}	v_{64}	v_{65}	v_{66}	v_{73}	v_{74}	v_{75}
$v_4:v_4$	v_{13}	v_{22}	v_6	v_{15}	v_{24}	v_5	v_{14}	v_{23}	v_7	v_{16}	v_{25}	v_9	v_{18}	v_{27}	v_8	v_{17}	v_{26}
$v_5:v_5$	v_{14}	v_{23}	v_4	v_{13}	v_{22}	v_6	v_{15}	v_{24}	v_9	v_{18}	v_{27}	v_8	v_{17}	v_{26}	v_7	v_{16}	v_{25}
$v_6:v_6$	v_{15}	v_{24}	v_5	v_{14}	v_{23}	v_4	v_{13}	v_{22}	v_8	v_{17}	v_{26}	v_7	v_{16}	v_{25}	v_9	v_{18}	v_{27}
$v_7:v_7$	v_{25}	v_{16}	v_8	v_{26}	v_{17}	v_9	v_{27}	v_{18}	v_4	v_{22}	v_{13}	v_5	v_{23}	v_{14}	v_6	v_{24}	v_{15}
$v_8:v_8$	v_{26}	v_{17}	v_9	v_{27}	v_{18}	v_7	v_{25}	v_{16}	v_6	v_{24}	v_{15}	v_4	v_{22}	v_{13}	v_5	v_{23}	v_{14}
$v_9:v_9$	v_{27}	v_{18}	v_7	v_{25}	v_{16}	v_8	v_{26}	v_{17}	v_5	v_{23}	v_{14}	v_6	v_{24}	v_{15}	v_4	v_{22}	v_{13}
$v_{13}:v_{13}$	v_{22}	v_4	v_{15}	v_{24}	v_6	v_{14}	v_{23}	v_5	v_{25}	v_7	v_{16}	v_{27}	v_9	v_{18}	v_{26}	v_8	v_{17}
$v_{14}:v_{14}$	v_{23}	v_5	v_{13}	v_{22}	v_4	v_{15}	v_{24}	v_6	v_{27}	v_9	v_{18}	v_{26}	v_8	v_{17}	v_{25}	v_7	v_{16}
$v_{15}:v_{15}$	v_{24}	v_6	v_{14}	v_{23}	v_5	v_{13}	v_{22}	v_4	v_{26}	v_8	v_{17}	v_{25}	v_7	v_{16}	v_{27}	v_9	v_{18}
$v_{16}:v_{16}$	v_7	v_{25}	v_{17}	v_8	v_{26}	v_{18}	v_9	v_{27}	v_{22}	v_{13}	v_4	v_{23}	v_{14}	v_5	v_{24}	v_{15}	v_6
$v_{17}:v_{17}$	v_8	v_{26}	v_{18}	v_9	v_{27}	v_{16}	v_7	v_{25}	v_{24}	v_{15}	v_6	v_{22}	v_{13}	v_4	v_{23}	v_{14}	v_5
$v_{18}:v_{18}$	v_9	v_{27}	v_{16}	v_7	v_{25}	v_{17}	v_8	v_{26}	v_{23}	v_{14}	v_5	v_{24}	v_{15}	v_6	v_{22}	v_{13}	v_4
$v_{22}:v_{22}$	v_4	v_{13}	v_{24}	v_6	v_{15}	v_{23}	v_5	v_{14}	v_{16}	v_{25}	v_7	v_{18}	v_{27}	v_9	v_{17}	v_{26}	v_8
$v_{23}:v_{23}$	v_5	v_{14}	v_{22}	v_4	v_{13}	v_{24}	v_6	v_{15}	v_{18}	v_{27}	v_9	v_{17}	v_{26}	v_8	v_{16}	v_{25}	v_7
$v_{24}:v_{24}$	v_6	v_{15}	v_{23}	v_5	v_{14}	v_{22}	v_4	v_{13}	v_{17}	v_{26}	v_8	v_{16}	v_{25}	v_7	v_{18}	v_{27}	v_9
$v_{25}:v_{25}$	v_{16}	v_7	v_{26}	v_{17}	v_8	v_{27}	v_{18}	v_9	v_{13}	v_4	v_{22}	v_{14}	v_5	v_{23}	v_{15}	v_6	v_{24}
$v_{26}:v_{26}$	v_{17}	v_8	v_{27}	v_{18}	v_9	v_{25}	v_{16}	v_7	v_{15}	v_6	v_{24}	v_{13}	v_4	v_{22}	v_{14}	v_5	v_{23}
$v_{27}:v_{27}$	v_{18}	v_9	v_{25}	v_{16}	v_7	v_{26}	v_{17}	v_8	v_{14}	v_5	v_{23}	v_{15}	v_6	v_{24}	v_{13}	v_4	v_{22}
$v_{28}:v_{28}$	v_{29}	v_{30}	v_{37}	v_{38}	v_{39}	v_{46}	v_{47}	v_{48}	v_{55}	v_{56}	v_{57}	v_{64}	v_{65}	v_{66}	v_{73}	v_{74}	v_{75}
$v_{29}:v_{29}$	v_{30}	v_{28}	v_{38}	v_{39}	v_{37}	v_{47}	v_{48}	v_{46}	v_{57}	v_{55}	v_{56}	v_{66}	v_{64}	v_{65}	v_{75}	v_{73}	v_{74}
$v_{30}:v_{30}$	v_{28}	v_{29}	v_{39}	v_{37}	v_{38}	v_{48}	v_{46}	v_{47}	v_{56}	v_{57}	v_{55}	v_{65}	v_{66}	v_{64}	v_{74}	v_{75}	v_{73}
$v_{37}:v_{37}$	v_{38}	v_{39}	v_{46}	v_{47}	v_{48}	v_{28}	v_{29}	v_{30}	v_{73}	v_{74}	v_{75}	v_{55}	v_{56}	v_{57}	v_{64}	v_{65}	v_{66}
$v_{38}:v_{38}$	v_{39}	v_{37}	v_{47}	v_{48}	v_{46}	v_{29}	v_{30}	v_{28}	v_{75}	v_{73}	v_{74}	v_{57}	v_{55}	v_{56}	v_{66}	v_{64}	v_{65}
$v_{39}:v_{39}$	v_{37}	v_{38}	v_{48}	v_{46}	v_{47}	v_{30}	v_{28}	v_{29}	v_{74}	v_{75}	v_{73}	v_{56}	v_{57}	v_{55}	v_{65}	v_{66}	v_{64}
$v_{46}:v_{46}$	v_{47}	v_{48}	v_{28}	v_{29}	v_{30}	v_{37}	v_{38}	v_{39}	v_{64}	v_{65}	v_{66}	v_{73}	v_{74}	v_{75}	v_{55}	v_{56}	v_{57}
$v_{47}:v_{47}$	v_{48}	v_{46}	v_{29}	v_{30}	v_{28}	v_{38}	v_{39}	v_{37}	v_{66}	v_{64}	v_{65}	v_{75}	v_{73}	v_{74}	v_{57}	v_{55}	v_{56}
$v_{48}:v_{48}$	v_{46}	v_{47}	v_{30}	v_{28}	v_{29}	v_{39}	v_{37}	v_{38}	v_{65}	v_{66}	v_{64}	v_{74}	v_{75}	v_{73}	v_{56}	v_{57}	v_{55}
$v_{55}:v_{55}$	v_{57}	v_{56}	v_{73}	v_{75}	v_{74}	v_{64}	v_{66}	v_{65}	v_{28}	v_{30}	v_{29}	v_{46}	v_{48}	v_{47}	v_{37}	v_{39}	v_{38}
$v_{56}:v_{56}$	v_{55}	v_{57}	v_{74}	v_{73}	v_{75}	v_{65}	v_{64}	v_{66}	v_{30}	v_{29}	v_{28}	v_{48}	v_{47}	v_{46}	v_{39}	v_{38}	v_{37}
$v_{57}:v_{57}$	v_{56}	v_{55}	v_{75}	v_{74}	v_{73}	v_{66}	v_{65}	v_{64}	v_{29}	v_{28}	v_{30}	v_{47}	v_{46}	v_{48}	v_{38}	v_{37}	v_{39}
$v_{64}:v_{64}$	v_{66}	v_{65}	v_{55}	v_{57}	v_{56}	v_{73}	v_{75}	v_{74}	v_{46}	v_{48}	v_{47}	v_{37}	v_{39}	v_{38}	v_{28}	v_{30}	v_{29}
$v_{65}:v_{65}$	v_{64}	v_{66}	v_{56}	v_{55}	v_{57}	v_{74}	v_{73}	v_{75}	v_{48}	v_{47}	v_{46}	v_{39}	v_{38}	v_{37}	v_{30}	v_{29}	v_{28}
$v_{66}:v_{66}$	v_{65}	v_{64}	v_{57}	v_{56}	v_{55}	v_{75}	v_{74}	v_{73}	v_{47}	v_{46}	v_{48}	v_{38}	v_{37}	v_{39}	v_{29}	v_{28}	v_{30}
$v_{73}:v_{73}$	v_{75}	v_{74}	v_{64}	v_{66}	v_{65}	v_{55}	v_{57}	v_{56}	v_{37}	v_{39}	v_{38}	v_{28}	v_{30}	v_{29}	v_{46}	v_{48}	v_{47}
$v_{74}:v_{74}$	v_{73}	v_{75}	v_{65}	v_{64}	v_{66}	v_{56}	v_{55}	v_{57}	v_{39}	v_{38}	v_{37}	v_{30}	v_{29}	v_{28}	v_{48}	v_{47}	v_{46}
$v_{75}:v_{75}$	v_{74}	v_{73}	v_{66}	v_{65}	v_{64}	v_{57}	v_{56}	v_{55}	v_{38}	v_{37}	v_{39}	v_{29}	v_{28}	v_{30}	v_{47}	v_{46}	v_{48}

El grupo E_3^* está representado mediante la siguiente tabla, la cual se compone de dos tablas más pequeñas:

*	ρ_{4}	ρ_{5}	ρ_{6}	ρ_{7}	ρ_{8}	ρ_{9}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}	ρ_{17}	ρ_{18}	ρ_{22}	ρ_{23}	ρ_{24}	ρ_{25}	ρ_{26}	ρ_{27}
ρ_{4}	ρ_{28}	ρ_{37}	ρ_{46}	ρ_{55}	ρ_{64}	ρ_{73}	ρ_{29}	ρ_{38}	ρ_{47}	ρ_{56}	ρ_{65}	ρ_{74}	ρ_{30}	ρ_{39}	ρ_{48}	ρ_{57}	ρ_{66}	ρ_{75}
ρ_{5}	ρ_{37}	ρ_{46}	ρ_{28}	ρ_{73}	ρ_{55}	ρ_{64}	ρ_{38}	ρ_{47}	ρ_{29}	ρ_{74}	ρ_{56}	ρ_{65}	ρ_{39}	ρ_{48}	ρ_{30}	ρ_{75}	ρ_{57}	ρ_{66}
ρ_{6}	ρ_{46}	ρ_{28}	ρ_{37}	ρ_{64}	ρ_{73}	ρ_{55}	ρ_{47}	ρ_{29}	ρ_{38}	ρ_{65}	ρ_{74}	ρ_{56}	ρ_{48}	ρ_{30}	ρ_{39}	ρ_{66}	ρ_{75}	ρ_{57}
ρ_{7}	ρ_{55}	ρ_{73}	ρ_{64}	ρ_{28}	ρ_{46}	ρ_{37}	ρ_{57}	ρ_{75}	ρ_{66}	ρ_{30}	ρ_{48}	ρ_{39}	ρ_{56}	ρ_{74}	ρ_{65}	ρ_{29}	ρ_{47}	ρ_{38}
ρ_{8}	ρ_{64}	ρ_{55}	ρ_{73}	ρ_{46}	ρ_{37}	ρ_{28}	ρ_{66}	ρ_{57}	ρ_{75}	ρ_{48}	ρ_{39}	ρ_{30}	ρ_{65}	ρ_{56}	ρ_{74}	ρ_{47}	ρ_{38}	ρ_{29}
ρ_{9}	ρ_{73}	ρ_{64}	ρ_{55}	ρ_{37}	ρ_{28}	ρ_{46}	ρ_{75}	ρ_{66}	ρ_{57}	ρ_{39}	ρ_{30}	ρ_{48}	ρ_{74}	ρ_{65}	ρ_{56}	ρ_{38}	ρ_{29}	ρ_{47}
ρ_{13}	ρ_{29}	ρ_{38}	ρ_{47}	ρ_{57}	ρ_{66}	ρ_{75}	ρ_{30}	ρ_{39}	ρ_{48}	ρ_{55}	ρ_{64}	ρ_{73}	ρ_{28}	ρ_{37}	ρ_{46}	ρ_{56}	ρ_{65}	ρ_{74}
ρ_{14}	ρ_{38}	ρ_{47}	ρ_{29}	ρ_{75}	ρ_{57}	ρ_{66}	ρ_{39}	ρ_{48}	ρ_{30}	ρ_{73}	ρ_{55}	ρ_{64}	ρ_{37}	ρ_{46}	ρ_{28}	ρ_{74}	ρ_{56}	ρ_{65}
ρ_{15}	ρ_{47}	ρ_{29}	ρ_{38}	ρ_{66}	ρ_{75}	ρ_{57}	ρ_{48}	ρ_{30}	ρ_{39}	ρ_{64}	ρ_{73}	ρ_{55}	ρ_{46}	ρ_{28}	ρ_{37}	ρ_{65}	ρ_{74}	ρ_{56}
ρ_{16}	ρ_{56}	ρ_{74}	ρ_{65}	ρ_{30}	ρ_{48}	ρ_{39}	ρ_{55}	ρ_{73}	ρ_{64}	ρ_{29}	ρ_{47}	ρ_{38}	ρ_{57}	ρ_{75}	ρ_{66}	ρ_{28}	ρ_{46}	ρ_{37}
ρ_{17}	ρ_{65}	ρ_{56}	ρ_{74}	ρ_{48}	ρ_{39}	ρ_{30}	ρ_{64}	ρ_{55}	ρ_{73}	ρ_{47}	ρ_{38}	ρ_{29}	ρ_{66}	ρ_{57}	ρ_{75}	ρ_{46}	ρ_{37}	ρ_{28}
ρ_{18}	ρ_{74}	ρ_{65}	ρ_{56}	ρ_{39}	ρ_{30}	ρ_{48}	ρ_{73}	ρ_{64}	ρ_{55}	ρ_{38}	ρ_{29}	ρ_{47}	ρ_{75}	ρ_{66}	ρ_{57}	ρ_{37}	ρ_{28}	ρ_{46}
ρ_{22}	ρ_{30}	ρ_{39}	ρ_{48}	ρ_{56}	ρ_{65}	ρ_{74}	ρ_{28}	ρ_{37}	ρ_{46}	ρ_{57}	ρ_{66}	ρ_{75}	ρ_{29}	ρ_{38}	ρ_{47}	ρ_{55}	ρ_{64}	ρ_{73}
ρ_{23}	ρ_{39}	ρ_{48}	ρ_{30}	ρ_{74}	ρ_{56}	ρ_{65}	ρ_{37}	ρ_{46}	ρ_{28}	ρ_{75}	ρ_{57}	ρ_{66}	ρ_{38}	ρ_{47}	ρ_{29}	ρ_{73}	ρ_{55}	ρ_{64}
ρ_{24}	ρ_{48}	ρ_{30}	ρ_{39}	ρ_{65}	ρ_{74}	ρ_{56}	ρ_{46}	ρ_{28}	ρ_{37}	ρ_{66}	ρ_{75}	ρ_{57}	ρ_{47}	ρ_{29}	ρ_{38}	ρ_{64}	ρ_{73}	ρ_{55}
ρ_{25}	ρ_{57}	ρ_{75}	ρ_{66}	ρ_{29}	ρ_{47}	ρ_{38}	ρ_{56}	ρ_{74}	ρ_{65}	ρ_{28}	ρ_{46}	ρ_{37}	ρ_{55}	ρ_{73}	ρ_{64}	ρ_{30}	ρ_{48}	ρ_{39}
ρ_{26}	ρ_{66}	ρ_{57}	ρ_{75}	ρ_{47}	ρ_{38}	ρ_{29}	ρ_{65}	ρ_{56}	ρ_{74}	ρ_{46}	ρ_{37}	ρ_{28}	ρ_{64}	ρ_{55}	ρ_{73}	ρ_{48}	ρ_{39}	ρ_{30}
ρ_{27}	ρ_{75}	ρ_{66}	ρ_{57}	ρ_{38}	ρ_{29}	ρ_{47}	ρ_{74}	ρ_{65}	ρ_{56}	ρ_{37}	ρ_{28}	ρ_{46}	ρ_{73}	ρ_{64}	ρ_{55}	ρ_{39}	ρ_{30}	ρ_{48}
ρ_{28}	ρ_{4}	ρ_{5}	ρ_{6}	ρ_{7}	ρ_{8}	ρ_{9}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}	ρ_{17}	ρ_{18}	ρ_{22}	ρ_{23}	ρ_{24}	ρ_{25}	ρ_{26}	ρ_{27}
ρ_{29}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{25}	ρ_{26}	ρ_{27}	ρ_{22}	ρ_{23}	ρ_{24}	ρ_{7}	ρ_{8}	ρ_{9}	ρ_{4}	ρ_{5}	ρ_{6}	ρ_{16}	ρ_{17}	ρ_{18}
ρ_{30}	ρ_{22}	ρ_{23}	ρ_{24}	ρ_{16}	ρ_{17}	ρ_{18}	ρ_{4}	ρ_{5}	ρ_{6}	ρ_{25}	ρ_{26}	ρ_{27}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{7}	ρ_{8}	ρ_{9}
ρ_{37}	ρ_{5}	ρ_{6}	ρ_{4}	ρ_{9}	ρ_{7}	ρ_{8}	ρ_{14}	ρ_{15}	ρ_{13}	ρ_{18}	ρ_{16}	ρ_{17}	ρ_{23}	ρ_{24}	ρ_{22}	ρ_{27}	ρ_{25}	ρ_{26}
ρ_{38}	ρ_{14}	ρ_{15}	ρ_{13}	ρ_{27}	ρ_{25}	ρ_{26}	ρ_{23}	ρ_{24}	ρ_{22}	ρ_{9}	ρ_{7}	ρ_{8}	ρ_{5}	ρ_{6}	ρ_{4}	ρ_{18}	ρ_{16}	ρ_{17}
ρ_{39}	ρ_{23}	ρ_{24}	ρ_{22}	ρ_{18}	ρ_{16}	ρ_{17}	ρ_{5}	ρ_{6}	ρ_{4}	ρ_{27}	ρ_{25}	ρ_{26}	ρ_{14}	ρ_{15}	ρ_{13}	ρ_{9}	ρ_{7}	ρ_{8}
ρ_{46}	ρ_{6}	ρ_{4}	ρ_{5}	ρ_{8}	ρ_{9}	ρ_{7}	ρ_{15}	ρ_{13}	ρ_{14}	ρ_{17}	ρ_{18}	ρ_{16}	ρ_{24}	ρ_{22}	ρ_{23}	ρ_{26}	ρ_{27}	ρ_{25}
ρ_{47}	ρ_{15}	ρ_{13}	ρ_{14}	ρ_{26}	ρ_{27}	ρ_{25}	ρ_{24}	ρ_{22}	ρ_{23}	ρ_{8}	ρ_{9}	ρ_{7}	ρ_{6}	ρ_{4}	ρ_{5}	ρ_{17}	ρ_{18}	ρ_{16}
ρ_{48}	ρ_{24}	ρ_{22}	ρ_{23}	ρ_{17}	ρ_{18}	ρ_{16}	ρ_{6}	ρ_{4}	ρ_{5}	ρ_{26}	ρ_{27}	ρ_{25}	ρ_{15}	ρ_{13}	ρ_{14}	ρ_{8}	ρ_{9}	ρ_{7}
ρ_{55}	ρ_{7}	ρ_{9}	ρ_{8}	ρ_{4}	ρ_{6}	ρ_{5}	ρ_{25}	ρ_{27}	ρ_{26}	ρ_{22}	ρ_{24}	ρ_{23}	ρ_{16}	ρ_{18}	ρ_{17}	ρ_{13}	ρ_{15}	ρ_{14}
ρ_{56}	ρ_{16}	ρ_{18}	ρ_{17}	ρ_{22}	ρ_{24}	ρ_{23}	ρ_{7}	ρ_{9}	ρ_{8}	ρ_{13}	ρ_{15}	ρ_{14}	ρ_{25}	ρ_{27}	ρ_{26}	ρ_{4}	ρ_{6}	ρ_{5}
ρ_{57}	ρ_{25}	ρ_{27}	ρ_{26}	ρ_{13}	ρ_{15}	ρ_{14}	ρ_{16}	ρ_{18}	ρ_{17}	ρ_{4}	ρ_{6}	ρ_{5}	ρ_{7}	ρ_{9}	ρ_{8}	ρ_{22}	ρ_{24}	ρ_{23}
ρ_{64}	ρ_{8}	ρ_{7}	ρ_{9}	ρ_{6}	ρ_{5}	ρ_{4}	ρ_{26}	ρ_{25}	ρ_{27}	ρ_{24}	ρ_{23}	ρ_{22}	ρ_{17}	ρ_{16}	ρ_{18}	ρ_{15}	ρ_{14}	ρ_{13}
ρ_{65}	ρ_{17}	ρ_{16}	ρ_{18}	ρ_{24}	ρ_{23}	ρ_{22}	ρ_{8}	ρ_{7}	ρ_{9}	ρ_{15}	ρ_{14}	ρ_{13}	ρ_{26}	ρ_{25}	ρ_{27}	ρ_{6}	ρ_{5}	ρ_{4}
ρ_{66}	ρ_{26}	ρ_{25}	ρ_{27}	ρ_{15}	ρ_{14}	ρ_{13}	ρ_{17}	ρ_{16}	ρ_{18}	ρ_{6}	ρ_{5}	ρ_{4}	ρ_{8}	ρ_{7}	ρ_{9}	ρ_{24}	ρ_{23}	ρ_{22}
ρ_{73}	ρ_{9}	ρ_{8}	ρ_{7}	ρ_{5}	ρ_{4}	ρ_{6}	ρ_{27}	ρ_{26}	ρ_{25}	ρ_{23}	ρ_{22}	ρ_{24}	ρ_{18}	ρ_{17}	ρ_{16}	ρ_{14}	ρ_{13}	ρ_{15}
ρ_{74}	ρ_{18}	ρ_{17}	ρ_{16}	ρ_{23}	ρ_{22}	ρ_{24}	ρ_{9}	ρ_{8}	ρ_{7}	ρ_{14}	ρ_{13}	ρ_{15}	ρ_{27}	ρ_{26}	ρ_{25}	ρ_{5}	ρ_{4}	ρ_{6}
ρ_{75}	ρ_{27}	ρ_{26}	ρ_{25}	ρ_{14}	ρ_{13}	ρ_{15}	ρ_{18}	ρ_{17}	ρ_{16}	ρ_{5}	ρ_{4}	ρ_{6}	ρ_{9}	ρ_{8}	ρ_{7}	ρ_{23}	ρ_{22}	ρ_{24}

*	ρ_{28}	ρ_{29}	ρ_{30}	ρ_{37}	ρ_{38}	ρ_{39}	ρ_{46}	ρ_{47}	ρ_{48}	ρ_{55}	ρ_{56}	ρ_{57}	ρ_{64}	ρ_{65}	ρ_{66}	ρ_{73}	ρ_{74}	ρ_{75}
ρ_{4}	ρ_{4}	ρ_{13}	ρ_{22}	ρ_{5}	ρ_{14}	ρ_{23}	ρ_{6}	ρ_{15}	ρ_{24}	ρ_{7}	ρ_{16}	ρ_{25}	ρ_{8}	ρ_{17}	ρ_{26}	ρ_{9}	ρ_{18}	ρ_{27}
ρ_{5}	ρ_{5}	ρ_{14}	ρ_{23}	ρ_{6}	ρ_{15}	ρ_{24}	ρ_{4}	ρ_{13}	ρ_{22}	ρ_{9}	ρ_{18}	ρ_{27}	ρ_{7}	ρ_{16}	ρ_{25}	ρ_{8}	ρ_{17}	ρ_{26}
ρ_{6}	ρ_{6}	ρ_{15}	ρ_{24}	ρ_{4}	ρ_{13}	ρ_{22}	ρ_{5}	ρ_{14}	ρ_{23}	ρ_{8}	ρ_{17}	ρ_{26}	ρ_{9}	ρ_{18}	ρ_{27}	ρ_{7}	ρ_{16}	ρ_{25}
ρ_{7}	ρ_{7}	ρ_{25}	ρ_{16}	ρ_{9}	ρ_{27}	ρ_{18}	ρ_{8}	ρ_{26}	ρ_{17}	ρ_{4}	ρ_{22}	ρ_{13}	ρ_{6}	ρ_{24}	ρ_{15}	ρ_{5}	ρ_{23}	ρ_{14}
ρ_{8}	ρ_{8}	ρ_{26}	ρ_{17}	ρ_{7}	ρ_{25}	ρ_{16}	ρ_{9}	ρ_{27}	ρ_{18}	ρ_{6}	ρ_{24}	ρ_{15}	ρ_{5}	ρ_{23}	ρ_{14}	ρ_{4}	ρ_{22}	ρ_{13}
ρ_{9}	ρ_{9}	ρ_{27}	ρ_{18}	ρ_{8}	ρ_{26}	ρ_{17}	ρ_{7}	ρ_{25}	ρ_{16}	ρ_{5}	ρ_{23}	ρ_{14}	ρ_{4}	ρ_{22}	ρ_{13}	ρ_{6}	ρ_{24}	ρ_{15}
ρ_{13}	ρ_{13}	ρ_{22}	ρ_{4}	ρ_{14}	ρ_{23}	ρ_{5}	ρ_{15}	ρ_{24}	ρ_{6}	ρ_{25}	ρ_{7}	ρ_{16}	ρ_{26}	ρ_{8}	ρ_{17}	ρ_{27}	ρ_{9}	ρ_{18}
ρ_{14}	ρ_{14}	ρ_{23}	ρ_{5}	ρ_{15}	ρ_{24}	ρ_{6}	ρ_{13}	ρ_{22}	ρ_{4}	ρ_{27}	ρ_{9}	ρ_{18}	ρ_{25}	ρ_{7}	ρ_{16}	ρ_{26}	ρ_{8}	ρ_{17}
ρ_{15}	ρ_{15}	ρ_{24}	ρ_{6}	ρ_{13}	ρ_{22}	ρ_{4}	ρ_{14}	ρ_{23}	ρ_{5}	ρ_{26}	ρ_{8}	ρ_{17}	ρ_{27}	ρ_{9}	ρ_{18}	ρ_{25}	ρ_{7}	ρ_{16}
ρ_{16}	ρ_{16}	ρ_{7}	ρ_{25}	ρ_{18}	ρ_{9}	ρ_{27}	ρ_{17}	ρ_{8}	ρ_{26}	ρ_{22}	ρ_{13}	ρ_{4}	ρ_{24}	ρ_{15}	υ_{6}	ρ_{23}	ρ_{14}	ρ_{5}
ρ_{17}	ρ_{17}	ρ_{8}	ρ_{26}	ρ_{16}	ρ_{7}	ρ_{25}	ρ_{18}	ρ_{9}	ρ_{27}	ρ_{24}	ρ_{15}	ρ_{6}	ρ_{23}	ρ_{14}	ρ_{5}	ρ_{22}	ρ_{13}	ρ_{4}
ρ_{18}	ρ_{18}	ρ_{9}	ρ_{27}	ρ_{17}	ρ_{8}	ρ_{26}	ρ_{16}	ρ_{7}	ρ_{25}	ρ_{23}	ρ_{14}	ρ_{5}	ρ_{22}	ρ_{13}	ρ_{4}	ρ_{24}	ρ_{15}	ρ_{6}
ρ_{22}	ρ_{22}	ρ_{4}	ρ_{13}	ρ_{23}	ρ_{5}	ρ_{14}	ρ_{24}	ρ_{6}	ρ_{15}	ρ_{16}	ρ_{25}	ρ_{7}	ρ_{17}	ρ_{26}	ρ_{8}	ρ_{18}	ρ_{27}	ρ_{9}
ρ_{23}	ρ_{23}	ρ_{5}	ρ_{14}	ρ_{24}	ρ_{6}	ρ_{15}	ρ_{22}	ρ_{4}	ρ_{13}	ρ_{18}	ρ_{27}	ρ_{9}	ρ_{16}	ρ_{25}	ρ_{7}	ρ_{17}	ρ_{26}	ρ_{8}
ρ_{24}	ρ_{24}	ρ_{6}	ρ_{15}	ρ_{22}	ρ_{4}	ρ_{13}	ρ_{23}	ρ_{5}	ρ_{14}	ρ_{17}	ρ_{26}	ρ_{8}	ρ_{18}	ρ_{27}	ρ_{9}	ρ_{16}	ρ_{25}	ρ_{7}
ρ_{25}	ρ_{25}	ρ_{16}	ρ_{7}	ρ_{27}	ρ_{18}	ρ_{9}	ρ_{26}	ρ_{17}	ρ_{8}	ρ_{13}	ρ_{4}	ρ_{22}	ρ_{15}	ρ_{6}	ρ_{24}	ρ_{14}	ρ_{5}	ρ_{23}
ρ_{26}	ρ_{26}	ρ_{17}	ρ_{8}	ρ_{25}	ρ_{16}	ρ_{7}	ρ_{27}	ρ_{18}	ρ_{9}	ρ_{15}	ρ_{6}	ρ_{24}	ρ_{14}	ρ_{5}	ρ_{23}	ρ_{13}	ρ_{4}	ρ_{22}
ρ_{27}	ρ_{27}	ρ_{18}	ρ_{9}	ρ_{26}	ρ_{17}	ρ_{8}	ρ_{25}	ρ_{16}	ρ_{7}	ρ_{14}	ρ_{5}	ρ_{23}	ρ_{13}	ρ_{4}	ρ_{22}	ρ_{15}	ρ_{6}	ρ_{24}
ρ_{28}	ρ_{28}	ρ_{29}	ρ_{30}	ρ_{37}	ρ_{38}	ρ_{39}	ρ_{46}	ρ_{47}	ρ_{48}	ρ_{55}	ρ_{56}	ρ_{57}	ρ_{64}	ρ_{65}	ρ_{66}	ρ_{73}	ρ_{74}	ρ_{75}
ρ_{29}	ρ_{29}	ρ_{30}	ρ_{28}	ρ_{38}	ρ_{39}	ρ_{37}	ρ_{47}	ρ_{48}	ρ_{46}	ρ_{57}	ρ_{55}	ρ_{56}	ρ_{66}	ρ_{64}	ρ_{65}	ρ_{75}	ρ_{73}	ρ_{74}
ρ_{30}	ρ_{30}	ρ_{28}	ρ_{29}	ρ_{39}	ρ_{37}	ρ_{38}	ρ_{48}	ρ_{46}	ρ_{47}	ρ_{56}	ρ_{57}	ρ_{55}	ρ_{65}	ρ_{66}	ρ_{64}	ρ_{74}	ρ_{75}	ρ_{73}
ρ_{37}	ρ_{37}	ρ_{38}	ρ_{39}	ρ_{46}	ρ_{47}	ρ_{48}	ρ_{28}	ρ_{29}	ρ_{30}	ρ_{73}	ρ_{74}	ρ_{75}	ρ_{55}	ρ_{56}	ρ_{57}	ρ_{64}	ρ_{65}	ρ_{66}
ρ_{38}	ρ_{38}	ρ_{39}	ρ_{37}	ρ_{47}	ρ_{48}	ρ_{46}	ρ_{29}	ρ_{30}	ρ_{28}	ρ_{75}	ρ_{73}	ρ_{74}	ρ_{57}	ρ_{55}	ρ_{56}	ρ_{66}	ρ_{64}	ρ_{65}
ρ_{39}	ρ_{39}	ρ_{37}	ρ_{38}	ρ_{48}	ρ_{46}	ρ_{47}	ρ_{30}	ρ_{28}	ρ_{29}	ρ_{74}	ρ_{75}	ρ_{73}	ρ_{56}	ρ_{57}	ρ_{55}	ρ_{65}	ρ_{66}	ρ_{64}
ρ_{46}	ρ_{46}	ρ_{47}	ρ_{48}	ρ_{28}	ρ_{29}	ρ_{30}	ρ_{37}	ρ_{38}	ρ_{39}	ρ_{64}	ρ_{65}	ρ_{66}	ρ_{73}	ρ_{74}	ρ_{75}	ρ_{55}	ρ_{56}	ρ_{57}
ρ_{47}	ρ_{47}	ρ_{48}	ρ_{46}	ρ_{29}	ρ_{30}	ρ_{28}	ρ_{38}	ρ_{39}	ρ_{37}	ρ_{66}	ρ_{64}	ρ_{65}	ρ_{75}	ρ_{73}	ρ_{74}	ρ_{57}	ρ_{55}	ρ_{56}
ρ_{48}	ρ_{48}	ρ_{46}	ρ_{47}	ρ_{30}	ρ_{28}	ρ_{29}	ρ_{39}	ρ_{37}	ρ_{38}	ρ_{65}	ρ_{66}	ρ_{64}	ρ_{74}	ρ_{75}	ρ_{73}	ρ_{56}	ρ_{57}	ρ_{55}
ρ_{55}	ρ_{55}	ρ_{57}	ρ_{56}	ρ_{73}	ρ_{75}	ρ_{74}	ρ_{64}	ρ_{66}	ρ_{65}	ρ_{28}	ρ_{30}	ρ_{29}	ρ_{46}	ρ_{48}	ρ_{47}	ρ_{37}	ρ_{39}	ρ_{38}
ρ_{56}	ρ_{56}	ρ_{55}	ρ_{57}	ρ_{74}	ρ_{73}	ρ_{75}	ρ_{65}	ρ_{64}	ρ_{66}	ρ_{30}	ρ_{29}	ρ_{28}	ρ_{48}	ρ_{47}	ρ_{46}	ρ_{39}	ρ_{38}	ρ_{37}
ρ_{57}	ρ_{57}	ρ_{56}	ρ_{55}	ρ_{75}	ρ_{74}	ρ_{73}	ρ_{66}	ρ_{65}	ρ_{64}	ρ_{29}	ρ_{28}	ρ_{30}	ρ_{47}	ρ_{46}	ρ_{48}	ρ_{38}	ρ_{37}	ρ_{39}
ρ_{64}	ρ_{64}	ρ_{66}	ρ_{65}	ρ_{55}	ρ_{57}	ρ_{56}	ρ_{73}	ρ_{75}	ρ_{74}	ρ_{46}	ρ_{48}	ρ_{47}	ρ_{37}	ρ_{39}	ρ_{38}	ρ_{28}	ρ_{30}	ρ_{29}
ρ_{65}	ρ_{65}	ρ_{64}	ρ_{66}	ρ_{56}	ρ_{55}	ρ_{57}	ρ_{74}	ρ_{73}	ρ_{75}	ρ_{48}	ρ_{47}	ρ_{46}	ρ_{39}	ρ_{38}	ρ_{37}	ρ_{30}	ρ_{29}	ρ_{28}
ρ_{66}	ρ_{66}	ρ_{65}	ρ_{64}	ρ_{57}	ρ_{56}	ρ_{55}	ρ_{75}	ρ_{74}	ρ_{73}	ρ_{47}	ρ_{46}	ρ_{48}	ρ_{38}	ρ_{37}	ρ_{39}	ρ_{29}	ρ_{28}	ρ_{30}
ρ_{73}	ρ_{73}	ρ_{75}	ρ_{74}	ρ_{64}	ρ_{66}	ρ_{65}	ρ_{55}	ρ_{57}	ρ_{56}	ρ_{37}	ρ_{39}	ρ_{38}	ρ_{28}	ρ_{30}	ρ_{29}	ρ_{46}	ρ_{48}	ρ_{47}
ρ_{74}	ρ_{74}	ρ_{73}	ρ_{75}	ρ_{65}	ρ_{64}	ρ_{66}	ρ_{56}	ρ_{55}	ρ_{57}	ρ_{39}	ρ_{38}	ρ_{37}	ρ_{30}	ρ_{29}	ρ_{28}	ρ_{48}	ρ_{47}	ρ_{46}
ρ_{75}	ρ_{75}	ρ_{74}	ρ_{73}	ρ_{66}	ρ_{65}	ρ_{64}	ρ_{57}	ρ_{56}	ρ_{55}	ρ_{38}	ρ_{37}	ρ_{39}	ρ_{29}	ρ_{28}	ρ_{30}	ρ_{47}	ρ_{46}	ρ_{48}

El grupo $\overline{F}_3^*$ está representado mediante la siguiente tabla, la cual se compone de dos tablas más pequeñas:

$*$	η_{4}	η_{5}	η_{6}	η_{7}	η_{8}	η_{9}	η_{13}	η_{14}	η_{15}	η_{16}	η_{17}	η_{18}	η_{22}	η_{23}	η_{24}	η_{25}	η_{26}	η_{27}
η_{4}	η_{55}	η_{73}	η_{64}	η_{28}	η_{46}	η_{37}	η_{56}	η_{74}	η_{65}	η_{29}	η_{47}	η_{38}	η_{57}	η_{75}	η_{66}	η_{30}	η_{48}	η_{39}
η_{5}	η_{73}	η_{64}	η_{55}	η_{37}	η_{28}	η_{46}	η_{74}	η_{65}	η_{56}	η_{38}	η_{29}	η_{47}	η_{75}	η_{66}	η_{57}	η_{39}	η_{30}	η_{48}
η_{6}	η_{64}	η_{55}	η_{73}	η_{46}	η_{37}	η_{28}	η_{65}	η_{56}	η_{74}	η_{47}	η_{38}	η_{29}	η_{66}	η_{57}	η_{75}	η_{48}	η_{39}	η_{30}
η_{7}	η_{28}	η_{37}	η_{46}	η_{55}	η_{64}	η_{73}	η_{30}	η_{39}	η_{48}	η_{57}	η_{66}	η_{75}	η_{29}	η_{38}	η_{47}	η_{56}	η_{65}	η_{74}
η_{8}	η_{46}	η_{28}	η_{37}	η_{64}	η_{73}	η_{55}	η_{48}	η_{30}	η_{39}	η_{66}	η_{75}	η_{57}	η_{47}	η_{29}	η_{38}	η_{65}	η_{74}	η_{56}
η_{9}	η_{37}	η_{46}	η_{28}	η_{73}	η_{55}	η_{64}	η_{39}	η_{48}	η_{30}	η_{75}	η_{57}	η_{66}	η_{38}	η_{47}	η_{29}	η_{74}	η_{56}	η_{65}
η_{13}	η_{56}	η_{74}	η_{65}	η_{30}	η_{48}	η_{39}	η_{57}	η_{75}	η_{66}	η_{28}	η_{46}	η_{37}	η_{55}	η_{73}	η_{64}	η_{29}	η_{47}	η_{38}
η_{14}	η_{74}	η_{65}	η_{56}	η_{39}	η_{30}	η_{48}	η_{75}	η_{66}	η_{57}	η_{37}	η_{28}	η_{46}	η_{73}	η_{64}	η_{55}	η_{38}	η_{29}	η_{47}
η_{15}	η_{65}	η_{56}	η_{74}	η_{48}	η_{39}	η_{30}	η_{66}	η_{57}	η_{75}	η_{46}	η_{37}	η_{28}	η_{64}	η_{55}	η_{73}	η_{47}	η_{38}	η_{29}
η_{16}	η_{29}	η_{38}	η_{47}	η_{57}	η_{66}	η_{75}	η_{28}	η_{37}	η_{46}	η_{56}	η_{65}	η_{74}	η_{30}	η_{39}	η_{48}	η_{55}	η_{64}	η_{73}
η_{17}	η_{47}	η_{29}	η_{38}	η_{66}	η_{75}	η_{57}	η_{46}	η_{28}	η_{37}	η_{65}	η_{74}	η_{56}	η_{48}	η_{30}	η_{39}	η_{64}	η_{73}	η_{55}
η_{18}	η_{38}	η_{47}	η_{29}	η_{75}	η_{57}	η_{66}	η_{37}	η_{46}	η_{28}	η_{74}	η_{56}	η_{65}	η_{39}	η_{48}	η_{30}	η_{73}	η_{55}	η_{64}
η_{22}	η_{57}	η_{75}	η_{66}	η_{29}	η_{47}	η_{38}	η_{55}	η_{73}	η_{64}	η_{30}	η_{48}	η_{39}	η_{56}	η_{74}	η_{65}	η_{28}	η_{46}	η_{37}
η_{23}	η_{75}	η_{66}	η_{57}	η_{38}	η_{29}	η_{47}	η_{73}	η_{64}	η_{55}	η_{39}	η_{30}	η_{48}	η_{74}	η_{65}	η_{56}	η_{37}	η_{28}	η_{46}
η_{24}	η_{66}	η_{57}	η_{75}	η_{47}	η_{38}	η_{29}	η_{64}	η_{55}	η_{73}	η_{48}	η_{39}	η_{30}	η_{65}	η_{56}	η_{74}	η_{46}	η_{37}	η_{28}
η_{25}	η_{30}	η_{39}	η_{48}	η_{56}	η_{65}	η_{74}	η_{29}	η_{38}	η_{47}	η_{55}	η_{64}	η_{73}	η_{28}	η_{37}	η_{46}	η_{57}	η_{66}	η_{75}
η_{26}	η_{48}	η_{30}	η_{39}	η_{65}	η_{74}	η_{56}	η_{47}	η_{29}	η_{38}	η_{64}	η_{73}	η_{55}	η_{46}	η_{28}	η_{37}	η_{66}	η_{75}	η_{57}
η_{27}	η_{39}	η_{48}	η_{30}	η_{74}	η_{56}	η_{65}	η_{38}	η_{47}	η_{29}	η_{73}	η_{55}	η_{64}	η_{37}	η_{46}	η_{28}	η_{75}	η_{57}	η_{66}
η_{28}	η_{4}	η_{5}	η_{6}	η_{7}	η_{8}	η_{9}	η_{13}	η_{14}	η_{15}	η_{16}	η_{17}	η_{18}	η_{22}	η_{23}	η_{24}	η_{25}	η_{26}	η_{27}
η_{29}	η_{22}	η_{23}	η_{24}	η_{16}	η_{17}	η_{18}	η_{4}	η_{5}	η_{6}	η_{25}	η_{26}	η_{27}	η_{13}	η_{14}	η_{15}	η_{7}	η_{8}	η_{9}
η_{30}	η_{13}	η_{14}	η_{15}	η_{25}	η_{26}	η_{27}	η_{22}	η_{23}	η_{24}	η_{7}	η_{8}	η_{9}	η_{4}	η_{5}	η_{6}	η_{16}	η_{17}	η_{18}
η_{37}	η_{5}	η_{6}	η_{4}	η_{9}	η_{7}	η_{8}	η_{14}	η_{15}	η_{13}	η_{18}	η_{16}	η_{17}	η_{23}	η_{24}	η_{22}	η_{27}	η_{25}	η_{26}
η_{38}	η_{23}	η_{24}	η_{22}	η_{18}	η_{16}	η_{17}	η_{5}	η_{6}	η_{4}	η_{27}	η_{25}	η_{26}	η_{14}	η_{15}	η_{13}	η_{9}	η_{7}	η_{8}
η_{39}	η_{14}	η_{15}	η_{13}	η_{27}	η_{25}	η_{26}	η_{23}	η_{24}	η_{22}	η_{9}	η_{7}	η_{8}	η_{5}	η_{6}	η_{4}	η_{18}	η_{16}	η_{17}
η_{46}	η_{6}	η_{4}	η_{5}	η_{8}	η_{9}	η_{7}	η_{15}	η_{13}	η_{14}	η_{17}	η_{18}	η_{16}	η_{24}	η_{22}	η_{23}	η_{26}	η_{27}	η_{25}
η_{47}	η_{24}	η_{22}	η_{23}	η_{17}	η_{18}	η_{16}	η_{6}	η_{4}	η_{5}	η_{26}	η_{27}	η_{25}	η_{15}	η_{13}	η_{14}	η_{8}	η_{9}	η_{7}
η_{48}	η_{15}	η_{13}	η_{14}	η_{26}	η_{27}	η_{25}	η_{24}	η_{22}	η_{23}	η_{8}	η_{9}	η_{7}	η_{6}	η_{4}	η_{5}	η_{17}	η_{18}	η_{16}
η_{55}	η_{7}	η_{9}	η_{8}	η_{4}	η_{6}	η_{5}	η_{25}	η_{27}	η_{26}	η_{22}	η_{24}	η_{23}	η_{16}	η_{18}	η_{17}	η_{13}	η_{15}	η_{14}
η_{56}	η_{25}	η_{27}	η_{26}	η_{13}	η_{15}	η_{14}	η_{16}	η_{18}	η_{17}	η_{4}	η_{6}	η_{5}	η_{7}	η_{9}	η_{8}	η_{22}	η_{24}	η_{23}
η_{57}	η_{16}	η_{18}	η_{17}	η_{22}	η_{24}	η_{23}	η_{7}	η_{9}	η_{8}	η_{13}	η_{15}	η_{14}	η_{25}	η_{27}	η_{26}	η_{4}	η_{6}	η_{5}
η_{64}	η_{8}	η_{7}	η_{9}	η_{6}	η_{5}	η_{4}	η_{26}	η_{25}	η_{27}	η_{24}	η_{23}	η_{22}	η_{17}	η_{16}	η_{18}	η_{15}	η_{14}	η_{13}
η_{65}	η_{26}	η_{25}	η_{27}	η_{15}	η_{14}	η_{13}	η_{17}	η_{16}	η_{18}	η_{6}	η_{5}	η_{4}	η_{8}	η_{7}	η_{9}	η_{24}	η_{23}	η_{22}
η_{66}	η_{17}	η_{16}	η_{18}	η_{24}	η_{23}	η_{22}	η_{8}	η_{7}	η_{9}	η_{15}	η_{14}	η_{13}	η_{26}	η_{25}	η_{27}	η_{6}	η_{5}	η_{4}
η_{73}	η_{9}	η_{8}	η_{7}	η_{5}	η_{4}	η_{6}	η_{27}	η_{26}	η_{25}	η_{23}	η_{22}	η_{24}	η_{18}	η_{17}	η_{16}	η_{14}	η_{13}	η_{15}
η_{74}	η_{27}	η_{26}	η_{25}	η_{14}	η_{13}	η_{15}	η_{18}	η_{17}	η_{16}	η_{5}	η_{4}	η_{6}	η_{9}	η_{8}	η_{7}	η_{23}	η_{22}	η_{24}
η_{75}	η_{18}	η_{17}	η_{16}	η_{23}	η_{22}	η_{24}	η_{9}	η_{8}	η_{7}	η_{14}	η_{13}	η_{15}	η_{27}	η_{26}	η_{25}	η_{5}	η_{4}	η_{6}

$*$:η_{28}	η_{29}	η_{30}	η_{37}	η_{38}	η_{39}	η_{46}	η_{47}	η_{48}	η_{55}	η_{56}	η_{57}	η_{64}	η_{65}	η_{66}	η_{73}	η_{74}	η_{75}
η_{4}:η_{4}	η_{22}	η_{13}	η_{5}	η_{23}	η_{14}	η_{6}	η_{24}	η_{15}	η_{7}	η_{25}	η_{16}	η_{8}	η_{26}	η_{17}	η_{9}	η_{27}	η_{18}
η_{5}:η_{5}	η_{23}	η_{14}	η_{6}	η_{24}	η_{15}	η_{4}	η_{22}	η_{13}	η_{9}	η_{27}	η_{18}	η_{7}	η_{25}	η_{16}	η_{8}	η_{26}	η_{17}
η_{6}:η_{6}	η_{24}	η_{15}	η_{4}	η_{22}	η_{13}	η_{5}	η_{23}	η_{14}	η_{8}	η_{26}	η_{17}	η_{9}	η_{27}	η_{18}	η_{7}	η_{25}	η_{16}
η_{7}:η_{7}	η_{16}	η_{25}	η_{9}	η_{18}	η_{27}	η_{8}	η_{17}	η_{26}	η_{4}	η_{13}	η_{22}	η_{6}	η_{15}	η_{24}	η_{5}	η_{14}	η_{23}
η_{8}:η_{8}	η_{17}	η_{26}	η_{7}	η_{16}	η_{25}	η_{9}	η_{18}	η_{27}	η_{6}	η_{15}	η_{24}	η_{5}	η_{14}	η_{23}	η_{4}	η_{13}	η_{22}
η_{9}:η_{9}	η_{18}	η_{27}	η_{8}	η_{17}	η_{26}	η_{7}	η_{16}	η_{25}	η_{5}	η_{14}	η_{23}	η_{4}	η_{13}	η_{22}	η_{6}	η_{15}	η_{24}
η_{13}:η_{13}	η_{4}	η_{22}	η_{14}	η_{5}	η_{23}	η_{15}	η_{6}	η_{24}	η_{25}	η_{16}	η_{7}	η_{26}	η_{17}	η_{8}	η_{27}	η_{18}	η_{9}
η_{14}:η_{14}	η_{5}	η_{23}	η_{15}	η_{6}	η_{24}	η_{13}	η_{4}	η_{22}	η_{27}	η_{18}	η_{9}	η_{25}	η_{16}	η_{7}	η_{26}	η_{17}	η_{8}
η_{15}:η_{15}	η_{6}	η_{24}	η_{13}	η_{4}	η_{22}	η_{14}	η_{5}	η_{23}	η_{26}	η_{17}	η_{8}	η_{27}	η_{18}	η_{9}	η_{25}	η_{16}	η_{7}
η_{16}:η_{16}	η_{25}	η_{7}	η_{18}	η_{27}	η_{9}	η_{17}	η_{26}	η_{8}	η_{22}	η_{4}	η_{13}	η_{24}	η_{6}	η_{15}	η_{23}	η_{5}	η_{14}
η_{17}:η_{17}	η_{26}	η_{8}	η_{16}	η_{25}	η_{7}	η_{18}	η_{27}	η_{9}	η_{24}	η_{6}	η_{15}	η_{23}	η_{5}	η_{14}	η_{22}	η_{4}	η_{13}
η_{18}:η_{18}	η_{27}	η_{9}	η_{17}	η_{26}	η_{8}	η_{16}	η_{25}	η_{7}	η_{23}	η_{5}	η_{14}	η_{22}	η_{4}	η_{13}	η_{24}	η_{6}	η_{15}
η_{22}:η_{22}	η_{13}	η_{4}	η_{23}	η_{14}	η_{5}	η_{24}	η_{15}	η_{6}	η_{16}	η_{7}	η_{25}	η_{17}	η_{8}	η_{26}	η_{18}	η_{9}	η_{27}
η_{23}:η_{23}	η_{14}	η_{5}	η_{24}	η_{15}	η_{6}	η_{22}	η_{13}	η_{4}	η_{18}	η_{9}	η_{27}	η_{16}	η_{7}	η_{25}	η_{17}	η_{8}	η_{26}
η_{24}:η_{24}	η_{15}	η_{6}	η_{22}	η_{13}	η_{4}	η_{23}	η_{14}	η_{5}	η_{17}	η_{8}	η_{26}	η_{18}	η_{9}	η_{27}	η_{16}	η_{7}	η_{25}
η_{25}:η_{25}	η_{7}	η_{16}	η_{27}	η_{9}	η_{18}	η_{26}	η_{8}	η_{17}	η_{13}	η_{22}	η_{4}	η_{15}	η_{24}	η_{6}	η_{14}	η_{23}	η_{5}
η_{26}:η_{26}	η_{8}	η_{17}	η_{25}	η_{7}	η_{16}	η_{27}	η_{9}	η_{18}	η_{15}	η_{24}	η_{6}	η_{14}	η_{23}	η_{5}	η_{13}	η_{22}	η_{4}
η_{27}:η_{27}	η_{9}	η_{18}	η_{26}	η_{8}	η_{17}	η_{25}	η_{7}	η_{16}	η_{14}	η_{23}	η_{5}	η_{13}	η_{22}	η_{4}	η_{15}	η_{24}	η_{6}
η_{28}:η_{28}	η_{29}	η_{30}	η_{37}	η_{38}	η_{39}	η_{46}	η_{47}	η_{48}	η_{55}	η_{56}	η_{57}	η_{64}	η_{65}	η_{66}	η_{73}	η_{74}	η_{75}
η_{29}:η_{29}	η_{30}	η_{28}	η_{38}	η_{39}	η_{37}	η_{47}	η_{48}	η_{46}	η_{57}	η_{55}	η_{56}	η_{66}	η_{64}	η_{65}	η_{75}	η_{73}	η_{74}
η_{30}:η_{30}	η_{28}	η_{29}	η_{39}	η_{37}	η_{38}	η_{48}	η_{46}	η_{47}	η_{56}	η_{57}	η_{55}	η_{65}	η_{66}	η_{64}	η_{74}	η_{75}	η_{73}
η_{37}:η_{37}	η_{38}	η_{39}	η_{46}	η_{47}	η_{48}	η_{28}	η_{29}	η_{30}	η_{73}	η_{74}	η_{75}	η_{55}	η_{56}	η_{57}	η_{64}	η_{65}	η_{66}
η_{38}:η_{38}	η_{39}	η_{37}	η_{47}	η_{48}	η_{46}	η_{29}	η_{30}	η_{28}	η_{75}	η_{73}	η_{74}	η_{57}	η_{55}	η_{56}	η_{66}	η_{64}	η_{65}
η_{39}:η_{39}	η_{37}	η_{38}	η_{48}	η_{46}	η_{47}	η_{30}	η_{28}	η_{29}	η_{74}	η_{75}	η_{73}	η_{56}	η_{57}	η_{55}	η_{65}	η_{66}	η_{64}
η_{46}:η_{46}	η_{47}	η_{48}	η_{28}	η_{29}	η_{30}	η_{37}	η_{38}	η_{39}	η_{64}	η_{65}	η_{66}	η_{73}	η_{74}	η_{75}	η_{55}	η_{56}	η_{57}
η_{47}:η_{47}	η_{48}	η_{46}	η_{29}	η_{30}	η_{28}	η_{38}	η_{39}	η_{37}	η_{66}	η_{64}	η_{65}	η_{75}	η_{73}	η_{74}	η_{57}	η_{55}	η_{56}
η_{48}:η_{48}	η_{46}	η_{47}	η_{30}	η_{28}	η_{29}	η_{39}	η_{37}	η_{38}	η_{65}	η_{66}	η_{64}	η_{74}	η_{75}	η_{73}	η_{56}	η_{57}	η_{55}
η_{55}:η_{55}	η_{57}	η_{56}	η_{73}	η_{75}	η_{74}	η_{64}	η_{66}	η_{65}	η_{28}	η_{30}	η_{29}	η_{46}	η_{48}	η_{47}	η_{37}	η_{39}	η_{38}
η_{56}:η_{56}	η_{55}	η_{57}	η_{74}	η_{73}	η_{75}	η_{65}	η_{64}	η_{66}	η_{30}	η_{29}	η_{28}	η_{48}	η_{47}	η_{46}	η_{39}	η_{38}	η_{37}
η_{57}:η_{57}	η_{56}	η_{55}	η_{75}	η_{74}	η_{73}	η_{66}	η_{65}	η_{64}	η_{29}	η_{28}	η_{30}	η_{47}	η_{46}	η_{48}	η_{38}	η_{37}	η_{39}
η_{64}:η_{64}	η_{66}	η_{65}	η_{55}	η_{57}	η_{56}	η_{73}	η_{75}	η_{74}	η_{46}	η_{48}	η_{47}	η_{37}	η_{39}	η_{38}	η_{28}	η_{30}	η_{29}
η_{65}:η_{65}	η_{64}	η_{66}	η_{56}	η_{55}	η_{57}	η_{74}	η_{73}	η_{75}	η_{48}	η_{47}	η_{46}	η_{39}	η_{38}	η_{37}	η_{30}	η_{29}	η_{28}
η_{66}:η_{66}	η_{65}	η_{64}	η_{57}	η_{56}	η_{55}	η_{75}	η_{74}	η_{73}	η_{47}	η_{46}	η_{48}	η_{38}	η_{37}	η_{39}	η_{29}	η_{28}	η_{30}
η_{73}:η_{73}	η_{75}	η_{74}	η_{64}	η_{66}	η_{65}	η_{55}	η_{57}	η_{56}	η_{37}	η_{39}	η_{38}	η_{28}	η_{30}	η_{29}	η_{46}	η_{48}	η_{47}
η_{74}:η_{74}	η_{73}	η_{75}	η_{65}	η_{64}	η_{66}	η_{56}	η_{55}	η_{57}	η_{39}	η_{38}	η_{37}	η_{30}	η_{29}	η_{28}	η_{48}	η_{47}	η_{46}
η_{75}:η_{75}	η_{74}	η_{73}	η_{66}	η_{65}	η_{64}	η_{57}	η_{56}	η_{55}	η_{38}	η_{37}	η_{39}	η_{29}	η_{28}	η_{30}	η_{47}	η_{46}	η_{48}

Definición 3.2. *Se define un álgebra de Dolph como el producto cruzado de una álgebra finita con un grupo finito. El grupo de unidades de esta álgebra se denomina grupo de Dolph.*

Los ejemplos obtenidos de álgebras de Dolph son E_2, F_2, E_3, F_3, I_3, J_3. Los grupos de unidades E_2^*, F_2^*, E_3^*, F_3^*, I_3^*, J_3^* son grupos de Dolph.

4. Conclusiones.

1. El discriminante Δ, dado en la Definición 3.1, visto como una aplicación del producto cruzado en el cuerpo correspondiente, preserva productos y unitarios. Así, es inmediato ver que $\overline{F}_3^*$ es un subgrupo de F_3^*.
2. Utilizando productos cruzados de álgebras de clarpolis con un grupo cíclico finito, obtenemos grupos finitos conmutativos E_2^*, F_2^*,E_3^*, $\overline{F}_3^*$, y grupos finitos no conmutativos I_3^*, $\overline{J}_3^*$.
3. Conforme al ejemplo [7, E 2.2.11)], el álgebra de grupo KG es una $K-$álgebra de Hopf, donde K es un cuerpo y G es un grupo. A partir de los cuerpos finitos $\mathbb{Z}_2$ y $\mathbb{Z}_3$, damos ejemplos de 4 clases de productos cruzados (ver [5, E 4.9, E 4.10]) según la trivialidad de la acción o del cociclo, como el producto tensorial E_2,E_3, el producto torcido F_2,F_3, el producto smash I_3, y el producto cruzado donde la acción y el cociclo son ambos no triviales, llamado J_3.
4. Por la Observación 2.2, para $\alpha \in \{1+x, 2+x, 1+2x, 2+2x\}$ se pueden obtener 4 productos cruzados y sus correspondientes grupos de unidades.
5. En este trabajo hemos obtenido 4 álgebras de Dolph con $\alpha \in \mathbb{Z}_3^*$ para el número primo 3. Luego, las álgebras de Dolph en el caso del número primo 5 quedan como materia de investigación futura.
6. Sea $\mathcal{J}_3$ el producto cruzado que se obtiene de J_3 cambiando el polinomio X^2 por el polinomio X^2+1 de la Observación 3.1. Se deja al lector estudiar $\mathcal{J}_3$ y compararlo con J_3.

Contribuición del autor. El artículo a sido desarrolla en su integridad por la autora: FC.

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